GAG 2013

Programme des mini-cours

Alastair Craw (Glasgow)

Titre : The McKay correspondence and Reid's recipe

Résumé : These lectures will describe the derived McKay correspondence in dimension three, focusing on examples arising in noncommutative toric geometry. In this context, one can illustrate the geometry of the derived equivalence in a precise way known as 'Reid's recipe'.

The classical case of the (toric) McKay correspondence in dimension three studies the relationship between the derived category of a crepant resolution of $\mathbf{C}^3/G$ and the derived category of the skew group algebra for a finite (abelian) subgroup G of $\mathbf{SL}(3, \mathbf{C})$. More generally, replacing $\mathbf{C}^3/G$ by any Gorenstein toric 3-fold leads to the study of dimer model algebras, and again, a derived equivalence linking the geometry of a crepant resolution and the derived category of modules over the noncommutative toric algebra can be described.

Philippe Eyssidieux (Grenoble)

Titre : Métriques de Kähler-Einstein canoniques, Flot de Kähler-Ricci et programme du Modèle Minimal

Résumé : Les progrès récents sur les solutions faibles des équations de Monge-Ampère complexes ont permis de généraliser considérablement les constructions de métriques de Kähler-Einstein issues de la solution de la conjecture de Calabi par Yau en autorisant que l'espace sous-jacent ait des singularités modérées, c'est à dire log-terminales.

On introduira ces équations elliptiques non-linéaires et leur variante parabolique, on décrira les diverses méthodes de résolution puis les constructions de métriques canoniques qu'elles permettent d'effectuer et qui constituent une géométrisation -au sens de la géométrie riemannienne- de certaines parties du programme des modèles minimaux.

Jun-Muk Hwang (KIAS, Séoul)

Titre : Lagrangian tori in projective symplectic manifolds

Résumé : One central problem regarding compact hyperkähler manifolds is to find a good condition for the existence of holomorphic or almost holomorphic fibrations on a compact hyperkähler manifold. Beauville proposed the question whether the existence of a Lagrangian torus A in a compact hyperkähler manifold M gives rise to such a fibration. It turns out that this question can be reduced to whether there exists a hypersurface in M disjoint from A. In this lecture, I will explain a joint work with Richard Weiss which gives an affirmative answer to the latter question. Our work employs two different tools : the theory of action-angle variables for algebraically completely integrable Hamiltonian systems and Wielandt's theory of subnormal subgroups. To apply these tools, we will introduc the notion of webs of submanfiolds and study their monodromy groups.

Alexander Kuznetsov (Moscou)

Titre : Derived categories of coherent sheaves in birational geometry

Résumé : I will discuss some interactions between derived categories of coherent sheaves and birational geometry, including construction of birational invariants and applications to resolutions of singularities. I will sketch a conjectural definition of the Griffiths component of the derived

category and its application for the understanding of birational properties of cubic forufolds. Also I will discuss categorical resolutions of singularities and different notions of crepancy in the categorical context.

Bertrand Toën (Montpellier)

Titre : Introduction to shifted symplectic structures

Résumé : The purpose of the series of lectures is to introduce the notion of shifted symplectic structures, a generalization of symplectic structures in the context of derived algebraic geometry. It will start with two motivating examples : the moduli stack of sheaves on CY manifolds, and representations of the fundamental group of a compact oriented manifold.

I will then present the formalism of derived algebraic geometry and explain some construction techniques (derived mapping spaces, derived fiber products). In the last part these techniques will be used to define shifted symplectic structures, and to state the main existence theorem : the derived mapping space (or stack) from an oriented object (e.g. a CY variety) towards a shifted symplectic target carries a canonical shifted symplectic structure.

The two motivating examples of sheaves on a CY, and of linear representations of fundamental groups, will be considered again as instances of applications of this general existence theorem.