Diophantine approximation and dynamical systems

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There are two fundamental results in metric Diophantine approximation: Khintchine's theorem and Jarnik's theorem. The former relates the size of the set of well approximable points, expressed in terms of Lebesgue measure, to the behaviour of a certain volume sum. The latter is a Hausdorff measure version of the former. We start by discussing these theorems and show that they are both in fact a simple consequence of the notion of "ubiquity". Also, we show that Khintchine's theorem without monotonicity (i.e the Duffin-Schaffer conjecture) implies Jarnik's theorem without monotonicity (i.e. the generalized Duffin-Schaffer conjecture). We then show how these theorems can be viewed from a dynamical point of view via Kleinian groups and rational maps. Also, the problem of Diophantine approximation on manifolds and Littlewood's conjecture will be discussed.