Counting integral points on homogeneous affine varieties and unipotent flows

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Finding asymptotic estimates for the number of integral points in large balls on affine algebraic subvarieties of $\mathbb{R}^n$ defined over $\mathbb{Q}$ is an important problem. It is typically addressed using the circle method. In some of the interesting questions, such a variety admits a transitive linear action of a reductive algebraic group, say G, defined over $\mathbb{Q}$ and the integral points turn out to be invariant under $G(\mathbb{Z})$. It was observed by Duke, Rudnick and Sarnak that one can reformulate the counting problem into equidistribution questions about subgroup actions on $G/G(\mathbb{Z})$.

Our aim in these talks is to show how one can use Ratner's theorems on unipotent flows on homogeneous spaces of Lie groups to study the reformulated problem. We shall give some examples where this approach works well.