# Éléments de mathématiques en sanskrit III
# Formation Doctorale

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# Meters in Sanskrit

The limbs of the *vedas*, called *vedāṅga* are

1. phonetics
2. knowledge of rites
3. grammar
4. etymology
5. prosody *chandaḥ*
6. astronomy.

Many meters exist since vedic period, for example the epics *Rāmāyaṇa* and *Mahābhārata* or *Bhagavadgītā* mostly use the *anuṣṭubh* meter.

**Example (opening of BG)**

*dharmakṣetre kurukṣetre*
*samavetā yuyutsavaḥ* /
*māmakaḥ pāṇdavāścaiva*
*kim akurva ta saṃjaya*//

*In the field of righteousness, in Kurukshetra, my sons and Pandu's sons gathered, eager to fight. What did they do, O Samjaya?*

# *Chandaḥśāstra*

- Study of meters of Sanskrit poetry by *Piṅgala* , 200-300 BCE. Improves many earlier authors. Specifically cited in CS : *Yāska, Māṇḍavya, Krauṣṭuki* etc.
- Influenced many many later authors till 14th century, one example is *Kedarabhaṭṭa* (11th).
- First european translation in 1863 (Albrecht Weber, Uber die Metrik der Inder).
- 315 *sūtras*
- 8 chapters The second chapter of Pirigala's Chandas Sutra introduces and discusses various aspects of the seven basic vedic metres: *gāyatrī, uṣṇik, anuṣṭubh, bṛhatī, paṅkti, triṣṭubh and jagatī,*
- 8.20-8.35 deal with combinatorics.
- Very cryptic verses. Difficult to undersand. The commentary of *Halāyudha* of Xth century *Mṛtasaṃjīvanī* (That which revives the dead) explains the text of CS from the point of vue of grammar and gives examples and applications. Must have been other commentaries before (lost).

Similar study by Marin Mersenne (1588-1648) in France.

# Building Block

Syllabic verse *Akṣarachandaḥ* depends on number of syllables ( *akṣara*) contained in a meter. For example, the *anuṣṭubh* meter contains 8 syllables per line.
A syllable is called a *laghu* (short), (denoted by L), if it consists of a short vowel followed by at most one consonant. , not followed by a conjunct consonant, an *anusvära* (a nasal) or a *visarga* (an aspirant). A syllable which is not a *laghu* is called a *guru* (long), (denoted by G). [1]

**Example (opening of BG )**

| | | | |
|---|---|---|---|
| 1. | *GGG* | *GLG* | *GG* |
| 2. | *LLG* | *GLG* | *LL* |
| 3. | *GLG* | *GLG* | *GL* |
| 4. | *LLG* | *LLG* | *LL* |

[1] These may be compared to the Greek syllables: *thesis* and *arsis*.

The *Vṛttachandaḥ* consists of 4 *pāda* (feet ), each foot having a specified sequence of long and short syllables.

**Example (*Abhijñāna Śakuntalaṃ* 4.8, *triṣṭubh*)**

*amī vediṃ paritaḥ klṛptadhiṣṇyāḥ*
*samidvantaḥ prānta saṃstīrṇa darbhāḥ* /
*apaghnanto duritaṃ havyagandhaiḥ*
*vairtānāstvaṃ vahnayaḥ pāvayantu* //

Benediction for *Śakuntala*

*May these sacrificial fires protect you, fires which are located around the sacrificial altar, which blaze with fuel, whose borders are strewn with grass and which drive out sins by the fragrance of sacrificial offerings,* Piṅgala's algorithms deal only with the *Vṛttachandaḥ*. These are of three types. ( in Chapter 5)

1. *Sama* (even) are the forms in which all four feet have the same sequence of short and long syllables.
2. In *ardhasama* (semi-even), the arrangement of short and long syllables in the odd feet is different from that in the even feet, but each pair has the same arrangement.
3. The forms which are neither *sama* or *ardhasama* are called *viṣama* (uneven).

Quantitative Verse: A *mātrā* is a time measure. A short syllable is assigned one *mātrā* while a long syllable is assigned two. (Chapter 4, CS) Example *āryā* meter. Later developped by *Prākṛt* prosodists.

# Formal Problems in Prosody

1. *Prastāra* [2] Spread

   Systematically lists all theoretically possible forms of a meter with a fixed number of syllables.

2. *Naṣṭaṃ* Annihilated, Lost

   Recovers the form of a meter when its serial number in the list is given.

3. *Uddiṣṭaṃ* Indicated

   Determines the serial number of a given form.

4. *Lagakriyā* Short-Long-Exercise

   Calculates the number of forms with a specified number of short syllables (or long syllables).

5. *Saṅkhyā* Count

   Calculates the total number of theoretically possible forms of a meter.

6. *Adhvayoga* Space measure

   Determines the amount of space needed to write down the entire list .

[2] No labels used by *Piṅgala*

*dvikau glau*
*miśrau ca*
*pṛthaglā miśrāḥ*
*vasavastrikāḥ*

CS 8.20-8.23

*A double G-L pair*
*And [the same] combined*
*The L combined separately*
*Eight triplets*

Gloss of Halāyudha *After having written the letter G above we will place the letter L below; this is the exhaustive extension for a single syllable.*
MS
Written as an array, For a verse in one syllable we put : $\begin{matrix} G \\ L \end{matrix}$ and we duplicate vertically for two syllables

$$\begin{matrix} G \\ L \\ G \end{matrix}$$

# Explanations

*The word AND makes us accumulate the above extension. .. The word 'combined ' means composed or joint.* [MS]

So we get

$$GG$$
$$LG$$
$$GL$$
$$LL$$

for two syllables.

*The word 'separately' means that in this matter of extension L and G are added separately. This gives meters of five, six (*gayatri*) and seven (*usnik*) and can be extended..* [MS]

So we first add $G$ after the constructed array and repeat with $L$.
For three syllables we get eight (*vasava*) possibilities.

**Example**

*We now construct a table for 1-5 syllables using the above algorithm.*

We note that this is a *recursive* algorithm. Some prosodists have used *iterative* algorithms, for example *Kedara* in *Vrttaratnakara* of 12th century.

# Explicit Construction

| | | | | | |
|---|---|---|---|---|---|
| $G$ | $GG$ | $GGG$ | $GGGG$ | $GGGGG$ | 1 |
| $L$ | $LG$ | $LGG$ | $LGGG$ | $LGGGG$ | 2 |
| (1) | $GL$ | $GLG$ | $GLGG$ | $GLGGG$ | 3 |
| | $LL$ | $LLG$ | $LLGG$ | $LLGGG$ | 4 |
| | (2) | $GGL$ | $GGLG$ | $GGLGG$ | 5 |
| | | $LGL$ | $LGLG$ | $LGLGG$ | 6 |
| | | $GLL$ | $GLLG$ | $GLLGG$ | 7 |
| | | $LLL$ | $LLLG$ | $LLLGG$ | 8 |
| | | (3) | $GGGL$ | $GGGLG$ | 9 |
| | | | $LGGL$ | $LGGLG$ | 10 |
| | | | $GLGL$ | $GLGLG$ | 11 |
| | | | $LLGL$ | $LLGLG$ | 12 |
| | | | $GGLL$ | $GGLLG$ | 13 |
| | | | $LGLL$ | $LGLLG$ | 14 |
| | | | $GLLL$ | $GLLLG$ | 15 |
| | | | $LLLL$ | $LLLLG$ | 16 |
| | | | (4) | $GGGGL$ | 17 |
| | | | | $LGGGL$ | 18 |
| | | | | $\vdots$ | |
| | | | | $LLLLL$ | 32 |
| | | | | (5) | |

# A later algorithm

In *Vṛttaratnākara*, *Kedāra* of 12th century gives an algorithm for creating a table of notes that is not recursive.

*Below the first G of the foot consisting of all G, put down L.*

*Repeatedly, make the rest same as what is above.*

*This is the procedure.*

*Supply G when missing.*

*[Continue] until all L created.*

## Example (3 syllables)

$GGG$
$LGG$
$GLG$
$LLG$
$GGL$
$LGL$
$GLL$
$LLL$

# Recover Lost Arrangement

Writing was often on sand and could be lost. Algorithm to recover $n^{th}$ row :

*If division by two, L*
*If added one, G*

CS-8.24, 8.25.

*Repeat G until the requisite number of syllables have been obtained*

MS

## Example

*$n = 6$*
*6 is even, put L and halve it*
*3 is odd, add 1 before halving, put G*
*2 is even, put L, halve it.*
*1 is odd, add 1 before halving, put G*
*For meter of length 3, we get LGL.*
*For lengths 4 and 5, we get LGLG and LGLGG .*

# Comparison with Binary Representation

Replacing $G$ by 0 and $L$ by 1, we get 101, and
$(5)_{10} = (101)_2$
The difference is because in this method 1 is represented as 000 and not 001. Shifted by one place.
It is possible to find the index from a given form by a reverse process (CS 8.26-8.27), binary to decimal.,

# Powers of 2

Number of meters of $n$ syllables.
Recursive method.

$$2^n = \begin{cases} (2^{n/2})^2, \text{n even} \\ 2.2^{n-1}, \ \text{n odd} \end{cases}$$

*If the exponent is divisible by 2, write '2'.*

*If not, subtract 1, write '0'.*

*If 0, multiply by 2, if 2 multiply by itself.*

CS 8.28-8.31

Notice that the generated string has to be taken in reverse order.
Complexity of algorithm is $O(\log_2 n)$ and not $O(n)$ .

## Example

*n=6.*
*6/2=3 , Write 2*
*3-1=2 , Write 0*
*2/2=1, Write 2*
*1 , Write 0.*

Now to get $2^6$, work in reverse order.

$$1 \times 2 = 2$$
$$2^2 = 4$$
$$4 \times 2 = 8$$
$$8^2 = 64$$

This is the standard method of calculating powers in later texts.

# Binomial Coefficients

*pare pūrṅaṃ*
*pare pūrṅaṃiti //*

CS 8.34-8.35

*for the next, completion*
*for the next, completion, and so on.*

This is cryptic and not understandable without commentary.

*First write one square cell at the top. Below it write two cells, extending half way on both sides. Below it three, below that again four, until desired number of places, such is the extension as a mountain. Begin by writing the number one in the first cell. In the other cells, we put the completion which comes from the two cells above it....*

MS

## Example (*Meru-Prastāra*)

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

- This generates the number of ways of choosing $r$ L-s from a total of $n$ symbols recursively. We get the binomial coefficient

$$\binom{n}{r} = \binom{n-1}{r} + \binom{n-1}{r-1}.$$

- Some controversy about Halayudha's commentary. How does he reach it?
- Probably because very similar algorithm was known by then, appears for example in *Bharata*'s *Natyāśāstra* of 2nd or 1st century BCE which calculates partial sums.

**Example (*Bharata's construction*)**

| | | | | | |
|---|---|---|---|---|---|
| 6 | | | | | |
| 5 | 15 | | | | |
| 4 | 10 | 20 | | | |
| 3 | 6 | 10 | 15 | | |
| 2 | 3 | 4 | 5 | 6 | |
| 1 | 1 | 1 | 1 | 1 | 1 |

- This looks much like the Pascal's triangle (XVIIe CE), rotated.
- *Virahanāṅka*'s *Vṛttajātisamuccaya* of VIIe century CE in *Prākṛta* evokes a *sūci prastāra* (needle) and a *meru prastāra*.
- *Agnipurāṇa* around 7th century includes reference to *meru prastāra*.
- *Mahavīra* around 850 CE calculated $\binom{n}{r}$ non-recursively.