Éléments de mathématiques en sanskrit II
Formation Doctorale

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Various Numerals

Brahmi		—	=	≡	+	ᔑ	ᔕ	ᔓ	ᔐ	ᔒ
Hindu	০	১	২	৩	৪	৫	৬	৭	৮	৯
Arabic	٠	١	٢	٣	٤	٥	٦	٧	٨	٩
Medieval	0	1	2	3	4	5	6	7	8	9
Modern	0	1	2	3	4	5	6	7	8	9

1. Verbal Forms
   - Primary Numbers *eka (1), dvi (2), tri (3), catur (4), pañca (5), ṣaṭ (6), sapta (7), aṣta (8) ,nava (9)*
   - Multiples of ten *daśa (10), vimśati (20).....*, Powers 10^4 *ayuta*
   - Compositions "seven hundred and twenty" *sapta śatāni vimśatiḥ* in *Ṛgveda* (1.164.11).
2. Decimal System
   - Usual Decimal System $\{1, 2, .., 9, 0\}$ Digit/Numeral is a symbol. Place value digit d in r^{th} place gives

     $$d \times 10^{r-1}$$

     Positional decimal in *Āryabhaṭīya* (499 CE).
   - Sanskrit "digit" is *aṅka* (literally, "mark") and the term for "place" is *sthāna*.

Bhūtasaṃkhyā

- words to express numbers
  bhūta : being, *saṃkhya* : number
- several words for one number
  very useful for verse form
- significant words
- Unit place first. (Read $\rightarrow$, though written $\leftarrow$)
  Maxim *aṅkānāṃ vāmato gatiḥ*

Example

2020 : *ākāśa-kara-nabhas-nayana*
sky-hand-sky-eye 0-2-0-2

Examples

0	*śunya, kha, ananta*
1	*indu, dharā, rūpa*
2	*netra, kara, yamala, pakṣa*
3	*agni* [1] *, kāla*
4	*veda, yuga*
5	*vāṇa*
6	*rasa* [2]
⋮	
10	*aṅgula, dik*
⋮	
24	*jina*

[1] 3 sacrificial fires *gārhapatya, āhavanīya, dakṣiṇa* householder's, oblation, southern

[2] 6 tastes of *Āyurveda* , *madhur* (sweet), *amla* (sour), *lavaṇa* (salty), *kaṭu* (bitter), *tikta* (bitter), *kaṣāya* (astringent)

- Well-understood within a context
- Sporadic use in ancient texts (*Srautasūtra* of *Kātāyana*)
- About 100 occurences in *Piṇgala* (about 300 BCE) . Decimals not used.
- With place value in *Yavanajātaka*, translation of a Greek text on astrology by *Sphujidhvaja*, who
  - Used *bindu* for zero in decimal. (Among the first occurences)
  - Ended text with *nārāyaṇaṅkendu mitābda*[3] 191 *śaka*, 269-270 CE.
- Transmitted to Tibet and South-East Asia. Earliest inscription in Cambodian *Śiva* temple *rasa-dasara-bāṇa* 526 śaka[4] as year of construction.

[3] *nārāyana* (1), *aṅka*, digits (9), *indu*, moon (1)
[4] 624-25 CE

π in word-numeral

Example

A value of π in *Kriyākramakarī*

mādhavācāryaḥ punaḥ atopyāsannatamām
paridhisaṅkhyāmuktavān |
vibudhanetragajāhihutāśanatriguṇavedabhavāraṇabāhavaḥ|
navanikharvamite vṛtivistare paridhimānamidam
jagadurbudhāḥ ||

$$\pi = \frac{2827433388233}{9 \times 10^{11}} = 3.141592653592$$

(correct to 11 places)
vibudha=33, netra=2, gaja=8, ahi=8, hutāśana=3, triguṇa=3, veda=4, bha=27, vāraṇa=8, bāhu=2, nava-nikharva$=9 \times 10^{11}$.

Kaṭapayādi

- Each consonant mapped to a digit $\{1, 2, \cdots, 9, 0\}$
- Not to consecutive numbers.
- Very popular in Kerala.
- The first complete definition can be dated to 1819 CE .

1	2	3	4	5	6	7	8	9	0
ka	kha	ga	gha	ṅa	ca	cha	ja	jha	ña
ṭa	ṭha	ḍa	ḍha	ṇa	ta	tha	da	dha	na
pa	pha	ba	bha	ma					
ya	ra	la	va	śa	ṣa	sa	ha		unattached vowels

- First datable occurence in 683 CE in a work of astronomy.
- The astronomical tables *zīj* in the arabic world used a similar system called *abjad* [5].

[5] *alif*, *bā*, *jim*, *dāl* in arabic

Words for longer numbers : Example *bhavati* (Madame) for 644.
Used for dates at the end of a work.

Example

$$sphītaṃ\ līlāvatārair\ idam\ iha\ kurutām\ āyurārogyasaukhyaṃ$$

Nārāyaṇīyam 100.11

The poem of *Bhaṭṭatiri*, ends by wishing long life, good health and happiness. But *āyurārogyasaukhyaṃ* can be read as a date in *kaṭapayādi*. We then have 17,12, 210 in *Kali-ahargaṇa* , the number of civil days from the beginning of the present Kali era. [6]

[6] 8th December 1586 CE, the Kali era begun at the sunrise on Friday 18th February 3102.

Application

- 72 rāgas of *Melakarta* described in Carnatic music in *Caturdaṇḍi prakāśikā* (1660) of the musicologist *Veṅkaṭamakhin*. These are root/ pure rāgas, also called *Janaka* rāgas, using all 8 notes, S, R, G, M, P, D, N, S, [7] therefore complete (*saṃpurna*) with the notes always in order (*krama*).
- Within a *rāga* the notes *S, P, S* are fixed.
- Octave has 12 semi-tones

$$S, R_1, R_2 = G_1, R_3 = G_2, G_3, M_1, M_2, P, D_1, D_2 = N_1, D_3 = N_2, N_3.^{8}$$

- The couples R_i, G_j and D_k, N_ℓ can be chosen in $\binom{4}{2} = 6$ ways each, M has 2 choices. $72 = 6 \times 6 \times 2$.
- First two consonants of name of *rāga* in *Kaṭapayādi* gives its number, according to which the *rāga*s are arranged.
- The arrangement is cyclic, each *cakra* (cycle) contains exactly one R_i, G_j and all possible D_k, N_ℓ. First 36 ragas contain M_1 and the second M_2 but are otherwise identical.
- The 12 *cakra*s named in *Bhūtasaṃkhya : indu, netra,..., āditya*.

[7] Comparable to *Do, Re, Mi, Fa, So, La, Ti, Do*

[8] Compare with $C, C^\sharp, D, D^\sharp, E, F, F^\sharp, G, G^\sharp, A, A^\sharp, B$

Prototype of Hashing system

- The Melakatra number n decides the notes.
  1. $c = \lceil \frac{n}{6} \rceil$ fixes R_i, G_j. After an initial value of (1,1) , $i = \lfloor \frac{c}{2} \rfloor$, $j = \begin{cases} c, & c \leq 3 \\ \lceil \frac{c}{2} \rceil, & c > 3 \end{cases}$
  2. $m = n \mod 6$ fixes D_k, N_ℓ

Example

rāga *Suryākāntaṃ*, (sa 7, ya 1), Number 17, *cakra* 3, (*agni*),

$$SR_1 G_3 M_1 PD_2 N_3$$

$c = \lceil \frac{17}{6} \rceil = 3, 17 = 5 \mod 6, m = 5, k = \lfloor \frac{5}{2} \rfloor = 2, \ell = \lceil \frac{5}{2} \rceil = 3$

rāga *Gamanāśrama*, Number 53=39+17, *cakra* 9,

$$SR_1 G_3 M_2 PD_2 N_3$$