The singularity spectrum of Lévy processes in multifractal time

Barral J.

We shall first explain in what subordinating a multifractal continuous time in a Lévy process is a natural operation. Then we shall explain the problems raised by the multifractal analysis of the resulting process and briefly indicate how to solve them. The results that will be presented were obtained in joint works with S. Seuret.

Fractal dimension of some random points generated by empirical and brownian oscillations

Berthet Ph.

We present limit theorems on increments of the empirical process and the Brownian motion at various scales. These rescaled increments are viewed as local processes indexed (i) by [0,1] or (ii) by a class of functions. The set of all the properly normalized increments defines a sequence which is almost surely relatively compact in the uniform topology. The limiting points are Strassen type functions in case (i) and the unit ball of the underlying reproducing kernel Hilbert space in case (ii). We first show how the size of the (possibly changing with time) set of the considered locations of the increments determines the normalizing sequences as well as, in case (i), the exact rates. Second we consider some random fractals defined as exceptional points generated by the thus depicted oscillation behavior of the empirical and Brownian processes. In case (i) we compute the Hausdorff-Besicovitch dimension of the random set of points where a limiting function of the increments is infinitely often reached at the best possible rate or intermediate rates. It turns out that at some small scales close to the non invariance principle the random fractals associated to functions from the boundary of the Strassen set are very different in the empirical case and in the Brownian case. In case (ii) it is not yet possible to deal with such Chung type rates hence we only compute Hausdorff dimensions as in Deheuvels and Mason (1995) in the situation considered by Mason (2004).

Convergence rates in the central limit theorem for the Kantorovich distance.

DEDECKER[1] J.

Let S_n be the partial sums of a stationary sequence of centered random variables, whose covariance series converges to σ^2. We give sufficient conditions "à la Gordin" under which the supremum of

$$|E(f(n^{-1/2}S_n)) - E(f(N(0,\sigma)))|$$

over the set of 1-Lipschitz functions is of order $n^{-1/2}$ or $\log(n)n^{-1/2}$. The optimal rate is reached for random variables having finite fourth moments. We also give conditions for random variables having finite third moments, which are expressed in terms of some weak dependence coefficients. These conditions are satisfied for some non irreducible Markov chains as well as some dynamical systems.

Nonparametric regression estimation for random fields

EL MACHKHOURI M.

We investigate the nonparametric estimation for regression in a fixed-design setting when the errors are given by a field of dependent random variables. Sufficient conditions for kernel estimators to converge are obtained. These estimators can attain the optimal rates of uniform convergence and the results apply to a large class of random fields which contains martingale-difference random fields and mixing random fields.

[1]joint work with RIO E.

Fractal aspects of stochastic processes

Falconer K.

The talk will review sample path properties of fractal Gaussian processes, in particular Brownian motion and its variants. We will discuss the local form and self-similarity of such processes and the ubiquity of the fractional Brownian motions, as well as higher dimensional analogues and the 'horizon problem'.

$U-$statistics of measure preserving transformation: central limit theorem

Gordin M.

Let T be an ergodic measure preserving transformation of probability space $(X, \mathcal{F}, P)$. For a suitable symmetric function $h : X^m \to \mathbb{R}$ the expression

$$U_n(h)(x) = \binom{n}{m}^{-1} \sum_{1 \le i_1 < \cdots < i_m \le n} h(T^{i_1}x, \ldots, T^{i_m}x), x \in X,$$

defines a sequence $(U_n(h))_{n \ge m}$ of functions on X called $U-$statistics of order m (with the kernel h). Some care is necessary to give sense to the above formula. Indeed, in this formula we only need to consider h restricted to some subsets of X^m which have the product measure zero. This does not make sense for a general measurable h or for $h \in L_2(X^m, \mathcal{F}^m, P^m)$, and we need to work within classes of more regular functions.

We consider the asymptotic behavior of $U-$statistics in two settings. The first one, when a $T-$invariant filtration is assumed to be specified, is suitable for applying martingale approximation and similar methods. However, this setup leads to rather restrictive assumptions concerning the kernel h. In particular, for $m = 2$ we deal with trace class kernels only. An equivalent assumption can be stated for general $m \ge 2$ in terms of the projective tensor power of $L_2(X, \mathcal{F}, P)$.

Looking for other classes of "admissible kernels" we try to relate the choice of the class with the dynamics represented by T. We are able to do so when

T is an ergodic surjective endomorphism of a compact abelian group X furnished with the Haar probability measure. Here we partially follow the approach in V.P. Leonov's classical paper of 1964.

In both setups we prove, under appropriate assumptions, the Central Limit Theorem for nondegenerate U-statistics.

Non differentiability in Euclidean geometry

Kahane J.-P.

The Dixmier point of a triangle will be defined, and then studied as a function of the data. We shall give a few results (Dixmier, Kahane, Nicolas) and mention open questions. All this is elementary geometry and classical analysis. The relation with stochastic processes and random fractals is not obvious.

A problem of estimation of analytic probability density function

Ibragimov I.

It is well known that an integer analytic function is determined by its values on any interval. Of course this unicity theorem is very unstable and small random perturbations in measurements can give rise large errors in the restoration of the function. As an example we consider a problem of restoration (estimation) of an integer probability density function on the base of iid observations taken into account only if they belong to a given bounded region.

Small Deviations of Fractional Processes with Respect to Fractal Measures

Lifshits[2] M.

We investigate Riemann–Liouville processes $R_H, H > 0$, and fractional Brownian motions B_H, $0 < H < 1$, and study their small deviation properties in the spaces $L_q([0,1],\mu)$. Of special interest are hereby thin (fractal) measures μ, i.e., those which are singular with respect to the Lebesgue measure. We describe the behavior of small deviation probabilities by numerical quantities of μ, called mixed entropy numbers, characterizing size and regularity of the underlying measure. For the particularly interesting case of self–similar measures the asymptotic behavior of the mixed entropy is evaluated explicitly. We also provide two–sided estimates for this quantity in the case of random measures generated by subordinators.

While the upper asymptotic bound for the small deviation probability is proved by purely probabilistic methods, the lower bound is verified by analytic tools concerning entropy and Kolmogorov numbers of Riemann–Liouville operators.

We also discuss multi-parametric versions of these results. They require non-trivial generalization of the notion of mixed entropy and generate a number of open questions.

Riemann-Liouville operators over fractal sets and applications to Gaussian processes

Linde W.

We investigate compactness properties of the Riemann-Liouville operator R_α of fractional integration when regarded as operator from $L_2[0,1]$ into $C(K)$, the space of continuous functions over a compact subset K in $[0,1]$. Of special interest are small sets K, i.e. those possessing Lebesgue measure zero (e.g. fractal sets). We describe the degree of compactness of R_α by the

[2]joint work with Linde W. and Shi Z.

size of K which is measured by covering properties. These results are applied for the investigation of the small ball behavior of certain Gaussian processes, as e.g. fractional Brownian motion or Riemann-Liouville processes, indexed by fractal sets.

Random Multifractals: History and Open Problems

Mandelbrot B.

On the weak invariance principle for stationary sequences under martingale-type conditions

Merlevede F.

The aim will be to present some recent results about the weak invariance principle for stationary sequences. For processes with short memory the theory of the weak invariance principle is very well fine tuned under various mixing conditions. The only problem is that, in some situations, the mixing conditions are not verified. We shall see how techniques using approximation by martingales can lead to innovative martingale-like conditions.

On the rate of approximation in limit theorems for linear processes

Paulauskas V.

In the talk we consider a linear process

$$X_t = \sum_{j=0}^{\infty} a_j \varepsilon_{t-j}, \tag{1}$$

where $\varepsilon_i, i \in \mathbf{Z}$ are i.i.d. random variables. We assume that random variables ε_i belong to the domain of attraction of a stable distribution (including Gaussian law) and we form

$$S_n := B_n^{-1} \sum_{t=1}^{n} X_t. \tag{2}$$

Under some additional requirements on random variables ε_i and coefficients a_i we get rates of approximation of distribution $P(S_n \leq x)$ by corresponding stable law. We distinguish the cases of short, long and negative memory for the process $X_t,\ t \geq 0$.

Limit theorems in Hölder topologies

RAČKAUSKAS A.

For rather general modulus of smoothness ρ, such as $\rho(h) = h^\alpha \log^\beta(c/h)$ we consider the Hölder space $H_\rho(B)$ of functions $[0,1] \to B$, where B is a separable Banach space. Using isomorphism between $H_\rho(B)$ and some sequence Banach space we study the central limit theorem for random elements in $H_\rho(B)$. Particularly, in this talk we discuss functional central limit theorem in Hölder topology. The talk is based on joint work with Ch. Suquet [1, 2].

References

[1] RAČKAUSKAS, A. and SUQUET, Ch. (2004). Central limit theorems in Hölder topologies for Banach space valued random fields. *Theory Probab. Appl.*, **49**, 77–92.

[2] RAČKAUSKAS, A. and SUQUET, Ch. (2004). Necessary and sufficient condition for the Hölderian functional central limit theorem. *Journal of Theoretical Probability* **17 (1)** 221–243.

Stationary tempered stable processes, their representations and classification

ROSINSKI J.

Tempered stable processes occur as result of random tempering of stable jumps. Tempered stable Lévy processes look locally like stable ones but globally they may approximate other stable processes of higher index or Brownian motions, depending on the intensity of tempering. This can be clearly seen from their shot noise representations.

In this talk we consider stationary tempered stable processes. We obtain their spectral representations which, surprisingly, are much simpler than the ones for stable processes. We relate both types of representations in this talk. Using spectral representations we derive classification and decompositions of stationary tempered stable processes and discuss examples.

Random rewards, Fractional Brownian local times and stable self-similar processes

Samorodnitsky[3] G.

We describe a new class of self-similar symmetric α-stable process with stationary increments arising as a large time scale limit in a situation when many users are earning random rewards or incurring random costs. The resulting models are different from the ones studied earlier both in their memory properties and smoothness of the sample paths.

Small deviations for moving average processes

Simon T.

We investigate the small ball probabilities under various norms for moving average processes defined by the convolution of a smooth function f : $]0, +\infty[\to \mathbb{R}$ with respect to a real $S\alpha S$ Lévy process. We show that the small ball exponent is uniquely determined by the norm and by the behaviour of f at zero. This improves on previous results of M. A. Lifshits and myself, where this was shown for f a power function (Riemann-Liouville processes). In the Gaussian case, the same generality as for RL processes is obtained with respect to the norms, thanks to a weak decorrelation inequality due to Li. In the more difficult Non-Gaussian case, we use a different method relying on comparison of entropy numbers, and restrict ourselves to Hölder and L_p-norms. This is joint work with Frank Aurzada (Jena).

2D Ising model and conformal invariance

Smirnov S.

We will discuss why 2D Ising model at critical temperature has a conformally invariant scaling limit.

[3]joint work with Cohen S.

The absolutely continuous transformations of the Poisson measure.

Smorodina N.V.

Let G be a metric space of the form $G = S \times [0, \infty)$, where S is a complete separable metric space. By $\mathcal{X} = \mathcal{X}(G)$ we denote the space of configurations on G. Let P be a Poisson measure on $\mathcal{X}$ with intensity measure Π of the form $\Pi(d\theta, dx) = \pi(d\theta)p(x)dx,\ \theta \in S, x \in [0, \infty)$, where π is a finite measure on S. We suppose that for every $\varepsilon > 0$ the function p is a strictly positive, bounded, continuous function on the set $[\varepsilon, \infty)$ and $\int_0^\infty \min(x, 1)p(x)dx < \infty$.

Let F be a measurable mapping $F : \mathcal{X} \to \mathcal{X}$ and PF^{-1} be a corresponding transformation of the measure P. We find conditions such that measure PF^{-1} (in general, nonpoissonian) is absolutely continuous with respect to the measure P and get the expression for the corresponding Radon-Nikodym derivative $\frac{dPF^{-1}}{dP}$.

As an example we consider the absolutely continuous, nonpoissonian transformation of the so-called gamma measure that is a Poisson measure with intensity measure of the form $\Pi(d\theta, dx) = \pi(d\theta)\frac{e^{-x}}{x}dx$.

Let P be a gamma measure and $F : \mathcal{X} \to \mathcal{X}$ be a mapping, defined by the formula

$$X = \{(\theta, x)\}_{(\theta,x)\in X} \mapsto F(X) = \{(\theta, xe^{f(\theta)F(X)})\}_{(\theta,x)\in X}.$$

Theorem. *Suppose that* $\int_S |f|d\pi < \infty$ *and the function* $\Phi : \mathcal{X} \to \mathbb{R}$ *is measurable, nonnegative and bounded. Also suppose that the function* Φ *satisfies the following conditions:*

1) for some $\varepsilon > 0$ *for every* $X \in \mathcal{X}$ $\quad \Phi(X) = \Phi(X \cap S \times [\varepsilon, \infty))$,

2) for every $k \in \mathbb{N}, (\theta_1, x_1), \dots, (\theta_k, x_k)$

$$1 + x_1 f(\theta_1)\frac{\partial \Phi_k}{\partial x_1} + \dots + x_k f(\theta_k)\frac{\partial \Phi_k}{\partial x_k} > 0$$

(here Φ_k *is a restriction of* Φ *on the set of* $k-$*pointed configurations).*

Then the measures P *and* PF^{-1} *are mutually absolutely continuous.*

Martingale approximations and the central limit theorem

Volny D.

In the talk, several recent results on martingale approximation of stationary sequences of random variables will be discussed. It will be shown that the existence of an approximating martingale depend on the choice of the filtration. In the case of a nonergodic measure the conditional central limit theorem can take place even if for all ergodic components, there is no convergence.

Random loops on surfaces

Werner W.

We will first discuss various descriptions of the unique measure on continuous self-avoiding loops in the plane that has certain conformal invariance properties. This will be related to critical percolation, Brownian motion and the conjectural scaling limit of self-avoiding walks. Then, we plan to explain how to construct a natural loop-valued process for which this measure is the stationnary measure.

Functional central limit theorem for multi-indexed summation process and its applications for panel data.

Zemlys V.

In the talk necessary and sufficient conditions will be established for the weak convergence of multi-indexed summation process to Brownian sheet in some Hölder spaces. Possible applications in econometric models for panel data will be discussed.

Strictly stable distributions on convex cones[4]

Zuyev S.

Using the LePage representation, a symmetric alpha-stable random element in a Banach space B with $0 < \alpha < 2$ can be represented as a sum of points of a Poisson process in B. This point process is union-stable, i.e. the union of its two independent copies coincides in distribution with the rescaled original point process. This shows that the classical definition of stable random elements is closely related to the union-stability property of point processes.

These concepts make sense in any convex cone C, i.e. in a semigroup equipped with multiplication by numbers, and lead to a construction of stable laws in general cones by means of the LePage series. Examples include compact sets with union operation, measures with convolution or with superposition operation, positive numbers with harmonic mean operation etc. We establish limit theorems for normalised sums of random elements with alpha-stable limit for $0 < \alpha < 1$ and deduce the LePage representation for strictly stable random vectors in these general cones.

By using the technique of harmonic analysis on semigroups we characterise distributions of alpha-stable random elements and show how possible values of the characteristic exponent alpha relate to the properties of the semigroup and the corresponding scaling operation, in particular, their distributivity properties.

[4]joint work with Youry Davydov and Ilya Molchanov.