Integrable systems and automorphic forms

February 19 to 22, 2019, Lille

Programme

Tuesday 19 February

9:00-10:00	Alexander Veselov
10:15-11:15	Alexandre Odesski
11:15-11:45	Coffee break
11:45-12:45	Ugo Bruzzo
13:00-14:00	Lunch
15:00-16:00	Valery Gritsenko
16:00-16:30	Coffee break
16:30-17:30	Armen Sergeev

Wednesday 20 February

9:00-10:00	Vincent Knibbeler
10:15-11:15	Jenya Ferapontov
11:15-11:45	Coffee break
11:45-12:45	Alexei Bolsinov
13:00-14:00	Lunch
15:00-16:00	Haowu Wang
16:00-16:30	Coffee break
16:30-17:30	Ann du Crest de Villeneuve

Thursday 21 February

9:00-10:00	Alexander Tikhomirov
10:15-11:15	Samuel Boissière
11:15-11:45	Coffee break
11:45-12:45	Vladimir Novikov
13:00-14:00	Lunch
15:00-16:00	Wadim Zudilin
16:00-16:30	Coffee break
16:30-17:00	Alexei Ivanov
17:10-17:40	Gaia Comaschi

19:30 Conference dinner

Friday 22 February

9:00-10:00	Fabien Cléry
10:15-11:15	Alessandra Sarti
11:15-11:45	Coffee break
11:45-12:45	Enrico Fatighenti
13:00-14:00	Lunch
14:30-15:30	Dmitri Adler
15:40-16:40	Volodya Roubtsov

ABSTRACTS

Dmitri Adler

Title: Constructive approach in the theory of Jacobi forms III: the root system D_n

Abstract: In 1992 K. Wirthmüller proved that the bigraded ring of the Weyl invariant weak Jacobi forms associated with an irreducible root system R (except E_8) is a free algebra over the ring of modular forms. However, his proof does not contain any explicit construction of generators of the polynomial ring which play an important role in the theory of flat coordinates of K. Saito. For the root system A_n, the generators were constructed by Bertola but his method does not work for the case of D_n. Moreover, Wirthmüller's proof is not very clear for D_n. In this talk, I will construct generators of the corresponding algebra in case of D_n root systems using only the classical Jacobi theta functions, modular forms and the modular differential operator. The talk will be based on joint work with Valery Gritsenko.

Samuel Boissière

Title: Some families of projective varieties uniformized by the 10-dimensional complex ball.

Abstract: In their famous paper, Allcock, Carlson and Toledo described the moduli space of cubic threefolds as the arithmetic quotient of the complement to a hyperplane arrangement in the 10-dimensional complex ball. I will present an interpretation of this moduli space as the one parametrizing a family of order three nonsymplectic automorphisms on hyperkähler manifolds deformation equivalent to the Hilbert square of two points on a K3 surface. This is a collaboration with Chiara Camere and Alessandra Sarti.

Alexey Bolsinov

Title: Symplectic invariants of integrable Hamiltonian systems: the case of degenerate singularities

Abstract: The nature of symplectic invariants for non-degenerate singularities of integrable Hamiltonian systems has been studied and clarified (both in the local and semi-local setting) in fundamentally important papers by Vey, Eliasson, Dufour, Toulet, Miranda, Zung and San Vu Ngoc. The talk is devoted to some new ideas and techniques that can be used for studying symplectic invariants of degenerate singularities.

As an example, I would like to discuss normal forms and symplectic invariants of parabolic orbits and cuspidal tori in integrable Hamiltonian systems with two degrees of freedom. Such singularities appear in many integrable systems in geometry and mathematical physics and can be considered as the simplest example of degenerate singularities. Some open problems will also be discussed.

Ugo Bruzzo

Title: Semistable Higgs bundles

Abstract: After a short introduction to Higgs bundles and their ubiquitous appearance in geometry, I will focus on some properties of Higgs bundles. I will mainly concentrate on two issues: a vanishing theorem for polystable Higgs bundles on Calabi–Yau varieties, and a conjecture about Higgs bundle that have the property of being semistable if pulled back to any curve.

Fabien Cléry

Title: Vector-valued Siegel modular forms of degree 2 and weight $(j, 2)$.

Abstract: Usual methods for determining dimensions of spaces of Siegel modular forms fail for small weights. In degree 2, this means that we do not have dimension formulas for weights (j, k) with $k < 3$. In this talk, we will propose some conjectural description of the spaces of Siegel modular forms of weight $(j, 2)$ on the level 2 principal congruence subgroup. We will provide some evidence for this conjectural description. This is a joint work with Gerard van der Geer and Gaëtan Chenevier.

Gaia Comaschi

Title: Linear spaces of skew-forms and moduli of instanton bundles

Abstract: Given a vector space W, a n-dimensional hyperweb of skew-forms is defined as a n-dimensional linear subspace of $\mathbb{P}(\Lambda^2 W^*)$, the projective space of skew-forms on W. In this talk I will treat the case where W is a (complex) vector space of dimension 6, presenting some results about 4-dimensional linear subspaces of $\mathbb{P}(\Lambda^2 W^*)$, with particular emphasis on those hyperwebs having generic rank equal to 4. I will explain how the study of the orbits under the action of $SL(6, \mathbb{C})$, more specifically the inspection of the stable ones, is motivated by the study of moduli of instanton bundles on cubic threefolds.

Ann du Crest de Villeneuve

Title: Tau functions of the Drinfeld–Sokolov hierarchies and Schur polynomials

Abstract: The Drinfeld–Sokolov hierarchies are families of integrable partial differential equations that are constructed via affine Kac–Moody algebras. The tau function is one single function that encodes a solution to a given hierarchy. In the spirit of Sato's expansion of the tau function of the KP hierarchy in terms of Schur polynomials and Plücker coordinates, I will show how to expand the tau function of any Drinfeld–Sokolov hierarchy over integer partitions via a natural generalization of Schur polynomials for arbitrary semisimple Lie algebras. We make use of the large-size limit Toeplitz determinants and the Sato–Segal–Wilson Grassmannian approach. We deduce a simple criterion for the polynomiality of the tau function. This is a joint work with Mattia Cafasso (Université d'Angers) and Di Yang (USTC, China).

Enrico Fatighenti

Title: Fano varieties of $K3$ type and IHS manifolds

Abstract: Subvarieties of Grassmannians (and especially Fano varieties) obtained from section of homogeneous vector bundles are far from being classified. A case of particular interest is given by the Fano varieties of K3 type, for their deep links with hyperkähler geometry. This talk will be mainly devoted to the construction of some new examples of such varieties. This is a work in progress with Giovanni Mongardi.

Evgeny Ferapontov

Title: Integrable Lagrangians and Picard modular forms

Abstract: We consider first-order Lagrangians such that the corresponding Euler–Lagrange equations are integrable (more precisely, belong to the class of dispersionless integrable systems). It is demonstrated that the generic integrable Lagrangian density is a Picard modular form of its arguments. Explicit parametrisation of the Lagrangian density by generalised hypergeometric functions of Appell type is obtained. Alternative representations and degenerations are also discussed. This talk is based on ongoing work with F. Clery and A. Odesskii, and notes of D. Zagier.

Valery Gritsenko

Title: Constructive approach in the theory of Jacobi forms I.

Abstract: In this talk and in the talks of my Ph. D. students H. Wang (Lille) and D. Adler (Moscow) we give a review of the recent results in theory of Jacobi forms in many variables and their applications.

The Chevalley type theorem for affine root systems is equivalent to the fact that the bigraded ring of the Weyl-invariant weak Jacobi forms for a classical root system R is a pure polynomial algebra. The corresponding generators play an important role in the theory of Frobenius varieties. This subject was developed by E. Looijenga, K. Saito, J. Bernstein, O. Schwarzman, K. Wirthmüller, B. Dubrovin, M. Bertola and others. The simplest case of the root system A_1 was considered by M. Eichler and D. Zagier in 1985. The corresponding generators are the normalised Jacobi theta-series and the elliptic genus of a Enriques surface. In the series of the talks we present similar results for different root systems. In the first talk I will give a general introduction in the subject and describe its relations with the theory of Borcherds products, geometry and mathematical physics.

Aleksei Ivanov

Title: Construction of rank 2 semistable sheaves on P3 using rational curves

Abstract: In this talk we describe new series of irreducible components of moduli schemes of coherent rank 2 semistable sheaves on $\mathbb{P}^3$. Generic points of these components correspond to sheaves which can be obtained from reflexive sheaves as elementary transformations along disjoint unions of rational curves and collections of points in $\mathbb{P}^3$. This series includes the components recently discovered by C. Almeida, M. Jardim and A. S. Tikhomirov as special case.

Vincent Knibbeler

Title: Hereditary Automorphic Lie Algebras

Abstract: Automorphic Lie algebras are spaces of equivariant meromorphic maps from a Riemann surface to a complex Lie algebra. Interest in these objects arose in the study of integrable PDEs in 1+1 dimensions. In this talk I will sketch recent advances in the structure theory of automorphic Lie algebras, as well as some open problems.

In particular, I will discuss the link between the class of hereditary automorphic Lie algebras and root cohomology. This link enables us to study automorphic Lie algebras which have been computationally out of reach so far, such as E_8.

Vladimir Novikov

Title: Integrability in 3 dimensions

Abstract: We consider the problem of detecting and classifying integrable partial differential (and difference) equations in 3D. Our approach is based on the observation that dispersionless limits of integrable systems in 3D possess infinitely many multi-phase solutions coming from the so-called hydrodynamic reductions. We consider a novel perturbative approach to the classification problem of dispersive equations. Based on the method of hydrodynamic reductions, we first classify integrable quasilinear systems which may (potentially) occur as dispersionless limits of soliton equations in 3D. To reconstruct dispersive deformations, we require that all hydrodynamic reductions of the dispersionless limit are inherited by the corresponding dispersive counterpart. This procedure leads to a complete list of integrable third and fifth order equations, which generalize the examples of Kadomtsev–Petviashvili, Veselov–Novikov and Harry Dym equations, as well as integrable Davey–Stewartson type equations, some of which are apparently new. We also consider the problem of dispersive deformations on the Lax representation level and thus show that our approach allows starting from the dispersionless Lax representations to construct the fully dispersive Lax pairs representing the fully dispersive integrable systems.

We extend this approach to the fully discrete case. Based on the method of deformations of hydrodynamic reductions, we classify discrete 3D integrable Hirota-type equations within various particularly interesting subclasses as well as a number of classification results of scalar differential-difference integrable equations including that of the intermediate long wave and Toda type.

Alexandre Odesski

Title: Poisson structures on loop spaces of $\mathbb{CP}^n$ and an r-matrix associated with the universal elliptic curve

Abstract: We construct a family of Poisson structures of hydrodynamic type on the loop space of $\mathbb{CP}^{n-1}$. This family is parametrized by the moduli space of elliptic curves or, in other words, by the modular parameter τ. This family can be lifted to a homogeneous Poisson structure on the loop space of $\mathbb{C}^n$ but in order to do that we need to upgrade the modular parameter τ to an additional field $\tau(x)$ with Poisson brackets $\{\tau(x), \tau(y)\} = 0, \quad \{\tau(x), z_a(y)\} = 2\pi i \; z_a(y) \; \delta'(x-y)$ where $z_1, ..., z_n$ are coordinates on $\mathbb{C}^n$. These homogeneous Poisson structures can be written in terms of an elliptic r-matrix of hydrodynamic type.

Volodya Roubtsov

Title: Non-commutative Calogero–Painlevé systems and Ruijsenaars duality revisited

Abstract: All Painlevé equations can be written as a time-dependent Hamiltonian system, and as such they admit a natural generalization to the case of several particles with an interaction of Calogero type (rational, trigonometric or elliptic). Recently, these systems of interacting particles have been proved to be relevant in the study of β-models. An almost two decade old open question by Takasaki asks whether these multi-particle systems can be understood as isomonodromic equations, thus extending the Painlevé correspondence. In this paper we answer in the affirmative by displaying explicitly suitable isomonodromic Lax pair formulations. The famous Ruijsenaars duality applies to the Calogero–Painlevé I, II, IV and the self-duality phenomena in these systems is discussed.

Alessandra Sarti

Title: Nikulin configurations on Kummer surfaces

Abstract: A Nikulin configuration is the data of 16 disjoint smooth rational curves on a $K3$ surface. According to results of Nikulin this means that the $K3$ surface is a Kummer surface and the abelian surface in the Kummer structure is determined by the 16 curves. A classical question of Shioda is about the existence of non isomorphic Kummer structures on the same Kummer K3 surface. The question was positively answered and studied by several authors, and it was shown that the number of non-isomorphic Kummer structures is finite, but no explicit geometric construction of such structures was given. In the talk I will show how to construct explicitely non isomorphic Kummer structures on generic Kummer $K3$ surfaces. This is a joint work with X. Roulleau.

Armen Sergeev

Title: Universal Teichmüller space and quantization of non-smooth string theory

Abstract: The universal Teichmüller space $\mathcal{T}$ is the space of quasisymmetric homeomorphisms of the unit circle S^1, i.e. homeomorphisms of S^1 extending to quasiconformal maps of the unit disk Δ, considered modulo Möbius transformations. It is a complex symplectic Banach manifold with the complex structure provided by embedding $\mathcal{T}$ into the complex Banach space of holomorphic quadratic differentials in a disk. The name of $\mathcal{T}$ is explained by the fact that all classical Teichmüller spaces, associated with compact Riemann surfaces, are contained in $\mathcal{T}$ as complex subvarieties. In the first part of my talk I shall present main complex geometric features of $\mathcal{T}$. My interest in this space is motivated by its relation to the string theory which I shall consider in the second part of my talk.

Alexander Tikhomirov

Title: New series of moduli spaces of stable bundles and sheaves on projective space

Abstract: We give an overview of recent results on the Gieseker–Maruyama moduli scheme $M = M(c_1, c_2, c_3)$ of rank 2 stable vector bundles and coherent sheaves with first Chern class $c_1 = 0$ or -1, second Chern class c_2, and third Chern class $c_3 \geq$ on the projective space $\mathbb{P}^3$. All currently known irreducible components of M for small values of c_2 and $c_3 \geq 0$ will be enumerated. We discuss the constructions of new series of components of M for arbitrary c_2. These are the resuts of our recent joint works with D. Markushevich, M. Jardim, A. Ivanov, D. Vasiliev, and others.

Alexander Veselov

Title: Chaos and integrability in $SL(2, \mathbb{R})$-geometry

Abstract: I will discuss the integrability of the geodesic flows on the threefolds admitting $SL(2, \mathbb{R})$-geometry in Thurston's sense. The main examples are the unit tangent bundles of genus $g > 1$ surfaces with hyperbolic metric. We show that the corresponding phase space T^*M^3 contains two open regions with integrable and chaotic behaviour respectively, explaining the automorphic character of the integrals.

The talk is based on a joint work with Alexey Bolsinov and Yiru Ye.

Haowu Wang

Title: Constructive approach in the theory of Jacobi forms II: the root system E_8

Abstract: For the classical root systems one defines Jacobi forms invariant under the Weyl group $W(R)$. In 1992, Wirthmüller proved that the bigraded ring of $W(R)$-invariant weak Jacobi forms is a polynomial algebra over the ring of $SL(2, \mathbb{Z})$ modular forms except the root system E_8. It is still an open problem how to extend Wirthmüller's theorem to the case $R = E_8$. The Weyl-invariant E_8 Jacobi forms have many applications in mathematics and physics, but very little has been known about their structure. In this talk, I will present an explicit description of the ring of $W(E_8)$-invariant Jacobi forms, which gives a solution to this old problem. First, I prove that the ring of $W(E_8)$-invariant weak Jacobi forms is NOT a polynomial algebra. Thus a Chevalley type theorem is NOT true for the root system E_8. Then I give a proper extension of the Chevalley type theorem to the case of affine root system E_8.

Wadim Zudilin

Title: The Mahler measure and Ising model

Abstract: The free energy of models in statistical mechanics reduces to calculation of a Mahler measure. We will explain a systematic approach to compute the latter through hypergeometric functions, and connections of the former with the values of L-functions of algebraic varieties and modular forms.