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RESEARCH STATEMENTS

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Statement of research

In the beginning of 2013 I got my Masters degree at the University of Helsinki with major in algebra and topology. My Masters thesis [A] is in PL-knot theory and concerns the construction of the Alexander invariant (a covering space) of a knot from a Seifert surface (orientable compact 2-manifold with a knot representative as its boundary) of the knot. Currently I am a PhD-student at the University of Helsinki and my advisor is Pekka Pankka.

My research topic is related to branched coverings (open discrete maps) and the Berstein-Edmonds construction [BE]: Assume that $f\colon N \to M$ is a branched covering between closed manifolds. Then there is such an open dense subset $N' \subset N$ that $g := f|N'\colon N' \to f[N']$ is a covering and $N \setminus N'$ does not locally separate N, see [V]. If the monodromy group of g is denoted ρ, then the Berstein-Edmonds construction gives a space X_f and a commutative diagram of branched coverings

$$\begin{tikzcd} & X_f \arrow[dl] \arrow[dr] & \\ N \arrow[rr, "f"'] & & M \end{tikzcd}$$

where $X_f \to M$ is an orbit map of the monodromy group ρ and $X_f \to N$ an orbit map of a subgroup $\tau < \rho$.

As a consequence of the compactness assumption on N and M all maps in the diagram above are finite-to-one maps. My first research questions are: Under which assumptions a Berstein-Edmonds construction exists for a branched covering $f\colon N \to M$ between manifolds with N not a closed manifold? What can be said about the branched covering f, the space X_f and the monodromy group ρ when the construction exists?

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Anabanti, Chimere Stanley University of Nigeria, Nsukka

Whitehead Algorithm for free groups

I just concluded my M.Sc research on "The Whitehead Algorithm for free groups" at the University of Warwick.

In 1936, Whitehead [3,4] used topological means to introduce a theorem which can be used to decide whether two elements of a finitely generated free group are equivalent under an automorphism of the group. Rapaport in [2] gave an algebraic proof of Whitehead's result. Higgins and Lyndon went further to simplify the result in [1].

In my M.Sc research, I established the corresponding algorithm and GAP program for classifying all minimal words of any given length in F_n up to equivalence. I went further to study the nature of representatives of the resulting equivalence classes of minimal words of lengths $2, 3, 4$ and 5 in F_n. The nature of such minimal words of lengths 2 and 3 is trivial. I concluded my M.Sc dissertation by establishing some conjectures on the nature of representatives of the resulting equivalence classes of minimal words of lengths 4 and 5 in F_n, and introducing new open problems.

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Expansion in finite groups

My research focuses on the construction of expander graphs. Expanders are sparse finite graphs which are "well-connected", in the sense that, for any partition of the vertex-set into two large components, the number of edges between the components will be high. Expanders have found diverse applications in theoretical computer science; group theory; number theory and hyperbolic geometry [1].

For many years, Cayley graphs of finite simple groups of Lie type have proved to be a rich source of examples of expanders. In a landmark paper [2], Bourgain and Gamburd showed that for an arbitrary finite group G, generated

by a subset S, the expansion of $Cay(G, S)$ may be quantified in terms of three pieces of data, namely: the minimal dimension of a non-trivial unitary representation of G; the growth-rate of product sets of an *arbitrary* generating set for G, and the extent to which the random walk on $Cay(G, S)$ avoids concentrating in proper subgroups of G. For finite simple groups of Lie type, the first of these three ingredients is taken care of by classical work, whereas the second (although difficult) is now a well-studied phenomenon, so most current research is focused on establishing the required "non-concentration" estimates in proper subgroups. The dependence in such estimates on the choice of the generating set S is a delicate matter: for instance, when $G = PSL_2(p)$, it is not known whether expansion occurs independent of both S and the prime p, but it *is* known that there is a density-one set of primes for which this is the case (though an explicit description of such a set of primes has never been given)[3].
Currently I am studying expansion in Cayley graphs of linear groups over finite fields of fixed characteristic. It has recently been shown that expansion in these groups occurs for generic generating sets S, but as yet few concrete examples are known [4]. Finding more such examples is the goal of my research.

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Sarah Bray Tufts University

Dynamics of nonstrictly convex divisible Hilbert geometries

We study compact quotients of discrete actions by $\Gamma < \mathrm{PGL}(n+1, \mathbb{R})$ acting on convex sets endowed with the Hilbert geometry. Any convex bounded domain Ω in n-dimensional real projective space, $\mathbb{RP}^n$, admits a Hilbert geometry as follows: take an affine slice of the cone over Ω in $\mathbb{R}^{n+1} \setminus \{0\}$. Given two points $x, y \in \Omega$, to define their Ω-distance we first extend the Euclidean line segment between x and y to the boundary $\partial\Omega$. Let a, b be the intersection points of $\overline{xy}$ and $\partial\Omega$. Then we define

$$d_\Omega(x, y) = \frac{1}{2}\big|\log[a, x, y, b]\big|$$

where $[a, x, y, b]$ is the standard Euclidean cross ratio:

$$[a, x, y, b] = \frac{|ay|}{|ax|} \frac{|bx|}{|by|}$$

It is a classical observation by Hilbert that d_Ω is a metric such that lines are geodesic, the boundary is at infinity, and projective transformations preserve the metric. Thus we can consider discrete subgroups of

$$\mathrm{Proj}(\Omega) = \{g \in \mathrm{PGL}(n+1, \mathbb{R}) \mid g\Omega = \Omega\}$$

acting by isometries on (Ω, d_Ω). We define Ω to be *divisible* when Ω admits a cocompact action by a discrete $\Gamma < \mathrm{Proj}(\Omega)$. Equivalently, Γ is said to *divide* Ω.

The first examples of divisible properly convex Ω arise from symmetric spaces. The classical case is an ellipsoid endowed with the Hilbert geometry, which is the Beltrami-Klein model for hyperbolic space $\mathbb{H}^n$. The other immediate examples are simplices and symmetric spaces of $SL(n, k)$ where k is $\mathbb{R}, \mathbb{C}$ or the quaternions or octonions (see [10] for more). Simplices endowed with the Hilbert geometry are isometric to $\mathbb{R}^n$ with a polygonal norm [6].

These examples provide early evidence for a relationship between regularity of the boundary and hyperbolicity of the acting group and the metric space. In [1] Benoist showed that strict convexity of divisible Ω is equivalent to C^1 regularity of $\partial\Omega$, δ-hyperbolicity of the metric space (Ω, d_Ω), and Gromov-hyperbolicity of Γ.

Benoist's work follows Benzecri's thesis in the 1970's. Benzecri proved a strong result on the PGL–orbits of properly convex Ω, and it follows that the only divisible Ω with boundary of class C^2 is the ellipsoid, a model for $\mathbb{H}^n$ [3].

Benoist also studied the dynamics of such strictly convex, divisible Ω in the following sense: any Hilbert geometry on a bounded convex domain Ω is compatible with a Finsler norm on the tangent space. We thus define the *line flow* on $T^1\Omega$ by flowing unit tangent vectors at unit speed along lines, which are geodesic. More specifically, the flow ϕ^t is defined by moving $v \in T^1\Omega$ along the geodesic line determined by v over time t.

Benoist proved in [1] that for a strictly convex, divisble Ω, the line flow on the compact quotient $M := \Omega/\Gamma$ is a mixing Anosov flow. The ergodic theory of line flows on strictly convex Ω is well understood.

In fact, it follows from the Anosov property of the flow that the boundary of a strictly convex divisible Ω must be $C^{1+\alpha}$ for some $0 < \alpha < 1$ [1,9], meaning the differential of the local graph $f : U \subset \partial\Omega \to \mathbb{R}$ is α-Hölder continuous:

$$|Df(x) - Df(y)| < |x - y|^\alpha$$

When Ω is not $\mathbb{H}^n$, the maximum such α is strictly less than 1, and explicit formulas for α_{max} are due to Guichard in [7]. Consequently, examples of divisible, strictly convex, non-symmetric Ω such as those constructed by Kac and Vinberg [8] have boundary of class $C^{1+\alpha}$.

Crampon [4,5] further studied the topological complexity of the line flow on compact projective structures with strictly convex universal cover Ω. He proved that topological entropy is bounded above by $n-1$ where n is the dimension of Ω, with equality only for the case where Ω is an ellipsoid [4]. He also computed directly that the Lyapunov exponents are determined by the convexity of the local graph of $\partial\Omega$ [5].

These same properties have yet to be explored for non-symmetric, non-strictly convex divisible Ω of higher dimensions. Benoist showed in [2] the existence of nontrivial examples of these Ω and their properties in dimension 3:

Theorem (Benoist): Let Ω be a properly convex open subset of $\mathbb{RP}^3$ divisible by discrete $\Gamma < \mathrm{Proj}(\Omega)$. If Ω is neither a polygon nor strictly convex, then

(1) There exist countably many open triangles $\triangle$ properly embedded in Ω, meaning $\partial\triangle \subset \partial\Omega$ but $\triangle \subset \Omega$.
(2) The triangles are disjoint in $\overline{\Omega}$.
(3) The collection vertices of the triangles are dense in $\partial\Omega$.
(4) The triangles are isolated in Ω.
(5) The compact quotient manifold Ω/Γ has an JSJ decomposition into atoroidal components connected by finitely many tori or Klein Bottle boundary components.
(6) The triangles project to boundary tori or Klein Bottles in the quotient.

The existence of such examples leads to natural dynamical questions:

- Is the flow topologically transitive? Mixing?
- Is there a measure of maximal entropy? If so, is it unique?
- Does the flow has some hyperbolicity properties (non-uniform hyperbolicity)?

There are some barriers to the study of these examples: the line flow here is not C^1 due to the C^0 continuity of $\partial\Omega$. Furthermore, there is no natural volume measure invariant under the flow.

The existence or non-existence of equilibrium measures is not obvious, but ergodicity with respect to some equilibrium state is hopeful given the presence of hyperbolic atoroidal components in the quotient.

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Julie Brönnimann Université de Neuchâtel

Geodesic growth of groups

Let $G = \langle S|R\rangle$ be a finitely presented group. For a word W over S, we denote by $\ell_S(W)$ the word length of W, and by $|W|_S$ the length of the element of G represented by W. A word W is *geodesic* if $\ell_S(W) = |W|_S$. For a group element $g \in G$, the length $|g|_S$ is the length of a shortest word W that represents g.

We define the *spherical growth function* of G to be the function $a_S : \mathbb{N} \to \mathbb{N}$ given by

$$a_S(n) \quad = \quad \#\{\, g \in G \mid |g|_S = n \,\},$$

and we define the *geodesic growth function* of G, with respect to the given presentation, to be $\gamma_S : \mathbb{N} \to \mathbb{N}$, where

$$\gamma_S(n) \quad = \quad \#\{\text{ geodesics of length } n \,\}.$$

A lot is known about the spherical growth function: for example, the growth type (such as polynomial, intermediate, or exponential) is independent of the generating set, Gromov showed that the groups with polynomial growth are exactly the virtually nilpotent ones, and the list of beautiful results goes on.

For the geodesic growth very little is known. One reason for the lack of results is that the geodesic growth is generating set dependent. My research is centered around products of groups. I would know how the number of geodesics in a direct product of groups, a free product, an amalgamated product and a HNN-extension grow. Particularly, I'm interested in the growth rate $\gamma(G, S) = \lim_{n\to\infty} \sqrt[n]{\gamma_S(n)}$. I would like to find bounds or a precise formula for the minimal growth rate, over all possible generating sets, of these products.

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Claire Burrin ETH Zurich

Rademacher functions and winding numbers

In [1], Atiyah identifies a number of functions on $\mathrm{SL}(2,\mathbb{Z})$ of different origins that however coincide, at least on hyperbolic elements. Motivated by this paper, Barge and Ghys pursued in [2] the unification of this family of functions via bounded cohomology, with the purpose of identifying various forms that the generator of the second cohomology group of $\mathrm{SL}(2,\mathbb{R})$ may take — Euler class, Rademacher function, Poincaré translation number, among others. Their insight was that the generators, albeit their very different nature (topologic, geometric, arithmetic, etc), share a common feature; they correspond to a quasimorphism on $\mathrm{SL}(2,\mathbb{Z})$.

There appears to be a similar unified approach to treat generalizations of the latter functions to some discrete subgroups of $\mathrm{PSL}(2,\mathbb{R})$. More precisely, consider a hyperbolic surface with a finite number of cusps, realized by the quotient of the Poincaré upper half-plane under the action of a particular Fuchsian group Γ. To each cusp, an analytical construction associates a non-vanishing section of the unit tangent bundle of the surface. This section is obtained by perturbing a generalization of the Dedekind η-function, whose logarithm is invariant under Γ-transformation up to an integer multiple of πi. This ambiguity defines an integer-valued quasimorphism on Γ that is the appropriate analog of the classical Rademacher function.

Such a Rademacher quasimorphism turns out to have a very simple geometric interpretation. It describes the winding number for closed geodesics parametrized by arc length, computed with respect to the non-vanishing section mentioned above. As such, the function has a finite sum representation in terms of a counting formula involving the values on the generators of the free group Γ. Moreover, it also coincides with the rotation quasimorphism introduced by Thomas Huber in his thesis [3]; a canonical rotation number living on the fundamental group of the unit tangent bundle that trivializes the Euler class.

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Ayala Byron Hebrew University, Jerusalem

Logic and geometry in free and hyperbolic groups

I am interested in geometric structures associated with free and hyperbolic groups, and in understanding the logical properties of such groups, using those structures.

On the way to solving Tarski's question about the first order theory of non-abelian free groups, Zlil Sela showed that to every limit group, and in particular to torsion-free (=tf) hyperbolic groups, one can associate a tower structure. Let us (very roughly) describe this structure for a torsion-free hyperbolic group G: the ground floor is a subgroup $G_0 \leq G$ (and one can choose G_0 so that it has no non-trivial tower structure over one of its subgroups. In this case G_0 is called the core of G). Each floor G_i is obtained from the previous one $G_{i-1} \leq G_i$ by gluing surfaces to G_{i-1} along their boundaries in a particular way, so that G_i is the fundamental group of the resulting graph of groups, and the last floor is G. Such a tower structure is made possible by the existence of a $JSJ-$decomposition for torsion-free hyperbolic groups. It turns out that those structures are very powerful in reflecting logical properties of the groups they represent. For example, Sela proved that a tf hyperbolic group is elementarily equivalent to its core, and that two tf hyperbolic groups are elementarily equivalent iff their cores are isomorphic. With Chloe Perin, we wish to understand how homogeneity of a tf hyperbolic group can be read out of a tower structure for it.

Going back to the free group (F), Sela proved the existence of an envelope for a definable set in F. This is a finite collection of towers, allowing one to understand which elements belong (or more commonly, do not belong) to the definable set, using sequences of morphisms from those towers to a free group. Together with Rizos Sklinos, we try to understand which definable algebraic structures a definable set in F can have. In particular, we can show that a definable set cannot be an abelian group.

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Caterina Campagnolo University of Geneva

Bounded cohomology and surface bundles

I am a third year PhD student, working under the supervision of Michelle Bucher.

The main topic of my thesis is the study of surface bundles via their characteristic classes, as defined by Morita (see [2]). A result of Gromov implies that these classes are bounded in degree $2(k+1)$, or in other words, that they can be represented by cocycles which are uniformly bounded. The question of boundedness for the remaining classes in degree $2k$ is open from degree 4 already.

One advantage of the theory of bounded cohomology, initiated by Gromov in the beginning of the 80's [1], is that good bounds for norms of cohomology classes naturally give rise to Milnor-Wood inequalities. One aspect of my work is thus to try to compute the norms of the characteristic classes of surface bundles, with as aim to produce new inequalities between classical invariants of surface bundles, such as the signature, the simplicial volume or the Euler characteristic of the total space of the bundle. At the moment I am focussing on surface bundles over surfaces, $\Sigma_h \hookrightarrow E \to \Sigma_g$, using the information provided by the first Morita class to approach these questions.

An important object in my research is the mapping class group of surfaces. In fact, characteristic classes of surface bundles are, in the universal case, cohomology classes of the mapping class group.

I am also interested in learning more about the Teichmüller space of surfaces. Indeed, it encodes much information on the configuration of the curves in a surface and it could help finding a geometric interpretation for representatives of cohomology classes of the mapping class group of the surface.

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Research statement

I am a Phd student at the University of Illinois at Chicago working with Alex Furman. My research interests include: ergodic theory, amenable groups, group actions and geometric group theory, random walks on finitely generated groups, and symbolic dynamics over non-amenable groups.

I am working on a project that fits under the heading of "The Zimmer Program", which has been described as the study of large group actions on compact manifolds. My project seeks to limit the possible actions of complicated amenable groups on compact manifolds. Precisely, suppose you have a finitely generated amenable group with the property that all of its finite index subgroups have no non-trivial homomorphisms into the reals. Suppose that this group acts by volume-preserving diffeomorphisms on a compact Riemannian manifold. Then I would like to show that there is a sequence of almost invariant Riemannian metrics for the action, in the sense that given epsilon and any finite subset of the acting group, there is a Riemannian metric that is epsilon-invariant under each of the elements from the given finite subset.

Juschenko and Monod have recently provided examples of finitely generated simple amenable groups, so the scope of our theorem would be nonempty.

One of the primary ingredients in the proof involves examining ergodic behavior of cocycles over amenable groups, which I find interesting and difficult. Motivation for the study of such behavior is to find an analog(s) of Kingmann's subadditive ergodic theorem for amenable groups, after Lindenstrauss. I have been working directly in these directions as well. For example, I am pursuing a subadditive ergodic theorem for cocycles over ergodic actions of $\mathbb{Z}^d$ and of nilpotent groups.

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Sandrine Caruso Université Rennes 1

Geometric and algebraic aspects of braid groups

The theory of braid groups is at the intersection of several areas of mathematics, especially algebra and geometry. The current research extends in each of these directions, leading to rich developments.

From a geometrical point of view, the braid group on n strands is seen as the mapping class group of a disc with n punctures, with boundary component. A braid can be represented by a curve diagram, that is to say, the image of a family of arcs attached to the disc, by the corresponding mapping class. In my thesis [1], I presented the algorithm of relaxations from the right, which, given a curve diagram, determines the braid from which it was obtained. This algorithm helps me to make the link between geometric properties of the curve diagram and algebraic properties of the braid word, allowing me to identify great powers of a generator as spirals in the curve diagram.

From an algebraic point of view, the braid group is the classical example of a Garside group. One of the objectives of current research in Garside theory is to obtain a polynomial time algorithm to solve the conjugacy problem in braid groups. For this, a possibility is to exploit the properties of some finite sets of conjugates of a braid, which are invariants of the conjugacy classes. One of my results ([1], [2]) concerns the size of one of these invariants, the super summit set: we construct a family of pseudo-Anosov braids whose super summit set has exponential size. González-Meneses had already established the similar result for a family of reducible braids [5]. These results implies that we cannot hope to solve the conjugacy problem in polynomial time through this set, and it is better to try to use smaller invariants. In the case of pseudo-Anosov braids, one may hope that the so-called sliding circuit set is more useful. With B. Wiest, we presented a polynomial time algorithm based on this last set which generically solves the conjugacy problem, that is to say, it solves it for a proportion of braids that tends exponentially fast to 1 as the length of the braid tends to infinity ([1], [4]). We also showed that, in a ball of the Cayley graph with generators the simple braids, a braid is generically pseudo-Anosov, which was a well-known conjecture for the specialists in Garside theory ([1], [4]).

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Positivity of the renormalized volume of quasi-Fuchsian hyperbolic 3-manifolds
–joint project with Sergiu Moroianu–

Let S be a closed surface of genus $g \geq 2$. A quasi-Fuchsian manifold is $S \times \mathbf{R}$ endowed with a complete hyperbolic metric. This metric induces a conformal structure on the boundary at infinity of $S \times \mathbf{R}$, which is the union of two copies of S. Conversely, by the Bers Simultaneous Uniformization Theorem, for any two points c_+, c_- in the Teichmüller space associated to S, there exists a unique quasi-Fuchsian manifold with c_+, c_- being, respectively, the induced conformal structures on its two boundaries at infinity.

Motivated by physicists needs, one aims to replace the quasi-Fuchsian infinite volume with a finite quantity. This is due to Schlenker [2] and Krasnov–Schlenker [1]. They introduce the notion of the renormalized volume, which turns out to be the log of a Kähler potential for the Weil–Peterson metric on the Teichmüller space. Their idea is to associate a finite number to each metric in the conformal structure at infinity of the quasi-Fuchsian manifold. This is done by foliating accordingly the two ends of the manifold, calculating the intermediate volume at time t determined by the foliation and then taking the finite part from the asymptotic expression of the intermediate volumes when $t \to \infty$. In general, the renormalized volume cannot be computed explicitly, given a metric in the conformal structure at infinity. This is because the definition of the renormalized volume really depends on the equidistant foliation associated to the metric in the conformal structure. Nevertheless, Krasnov–Schlenker prove that, under variations of the metric in its conformal class that keep the area of boundary at infinity fixed, the extremum of the renormalized volume occurs only for the metrics at infinity of constant negative curvature. Furthermore, by considering the second variation, those critical points are local maxima. But a natural question arises. Is the renormalized volume corresponding to the hyperbolic metric in the conformal class a positive quantity? In a joint project with Sergiu Moroianu we aim to study whether the renormalized volume is indeed a 'genuine volume' quantity for a quasi-Fuchsian manifold.

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Antoine Clais Université Lille 1

Combinatorial Loewner Property on boundaries of right-angled hyperbolic buildings.

We are working on groups called *graph products*, acting on *right-angled hyperbolic buildings*. The definition of graph products generalizes the classes of right-angled Coxeter and Artin groups. Let $\mathcal{G}$ be a simplicial graph. We denote $\mathcal{G}^{(0)} = \{v_1, \cdots, v_n\}$. To each vertex v_i we associate an element q_i of the set $\mathbb{N}\backslash\{0,1\} \cup \{\infty\}$. The *graph product* $\Gamma = (\mathcal{G}, \{q_i\}_{i=1\cdots n})$ is defined by the following presentation :

$$\Gamma = \langle s_1, \cdots, s_n | s_i^{q_i} = 1 \text{ and } [s_i, s_j] = 1 \text{ if } v_i \text{ adjacent to } v_j \text{ in } \mathcal{G} \rangle\,.$$

We now assume that Γ is Gromov hyperbolic and we denote $\partial\Gamma$ its visual boundary. Using a construction due to Davis and Moussong, Γ acts on a *right-angled hyperbolic building* Σ. This action is transitive and free on the set of chambers of Σ and $\partial\Sigma \simeq \partial\Gamma$.

We would like to find some examples of boundaries of such groups satisfying the *Combinatorial Loewner Property* (*CLP*). To define this property we first need to define the *combinatorial modulus* which is essentially a measure (in a weak sense) on the set of all the curves in $\partial\Gamma$. Then, denoting by A and B two connected compact and disjoint subsets of $\partial\Gamma$ the CLP is satisfied if : *the combinatorial modulus of the set of all curves joining A to B is controlled by $\Delta(A,B)$ the relative distance between A and B,*

$$\Delta(A,B) = \frac{\operatorname{dist}(A,B)}{\min\{\operatorname{diam}(A), \operatorname{diam}(B)\}}.$$

This may be proved thanks to the large amount of symmetries that the building structure and the action of Γ induce on the curves of $\partial\Gamma$.

Recently Bourdon and Kleiner found a necessary condition for the boundary of a hyperbolic Coxeter group to satisfy the CLP. In their paper they present lots of examples of Coxeter groups that verify this condition and they use the CLP to give a new proof of the Cannon conjecture for Coxeter groups. Their criterion may be adapted to the case of graph products. Now we are looking for examples of graph products that satisfy the criterion. Then finding the CLP on the boundary of some hyperbolic buildings could lead to show some rigidity results on the buildings.

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Contracting boundaries

While boundaries of hyperbolic spaces are invariant under quasi-isometry, Croke and Kleiner showed this is not the case in CAT(0) spaces. If one restricts attention to rays with hyperbolic-like behavior, so-called "contracting" rays, then one can define a boundary on CAT(0) spaces that is a quasi-isometry invariant [1]. (Given a fixed constant D, a geodesic γ is said to be D-*contracting* if $\forall x, y \in X$, $d_X(x, y) < d_X(x, \pi_\gamma(x)) \Rightarrow d_X(\pi_\gamma(x), \pi_\gamma(y)) < D$ where $\pi_\gamma(x)$ is the nearest point projection of x onto γ.) My research interests lie in using this boundary to identify quasi-isometry classes of groups. Currently, I am looking into what the "contracting boundary" can tell us about about the quasi-isometry classes of right-angled Coxeter groups.

I have also generalized the contracting boundary so that it is defined for any geodesic metric space, not just CAT(0) spaces. In the case of this boundary, I looked at quasi-geodesic rays that were "Morse"-quasi-geodesic rays. (A quasi-geodesic γ is M-*Morse* if for any constants $K \geq 1$, $L \geq 0$, there is a constant $M = M(K, L)$, such that for every (K, L)-quasi-geodesic σ with endpoints on γ, we have $\sigma \subset N_M(\gamma)$.) In the case of a CAT(0) space, Morse and contracting rays are equivalent and this boundary is homeomorphic to the contracting boundary. In the case of a hyperbolic space, this Morse boundary is homeomorphic to the visual boundary.

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Groups and deformation in complex hyperbolic geometry

The subjects that I am interested in fall within the context of hyperbolic geometry, which could refer in particular to the study of the Riemannian manifolds modelled on hyperbolic space $\mathbb{H}^n_{\mathbb{R}}$ and more generally to that of hyperbolic metric spaces in the sense of Gromov. Another class of objects appears in complex hyperbolic geometry, which proves quite different from a mere translation of real hyperbolic space into the language of complex numbers.

In the continuous world of Lie groups, one may try to deform a lattice into another while estimating to which extent a specific property is preserved. For instance, a lattice of the group of orientation-preserving isometries of $\mathbb{H}^3_{\mathbb{R}}$ is quasi-Fuchsian whenever it is obtained from a Fuchsian group by a quasi-conformal deformation. Although one may decide whether a lattice is quasi-Fuchsian in special cases, raising the hope that a more general answer exists, there is still no such method.

The answer to a special case was part of the proof of the surface subgroup conjecture by Kahn and Markovic [1]. However the ingredients do not seem peculiar to real hyperbolic geometry nor to dimension 3. For example, the variational formula of Wolpert-Kerckhoff-Series [2] admits a counterpart in $\mathbb{H}^2_{\mathbb{C}}$ [3]. On the other hand, some arguments belong in the discrete world of Gromov hyperbolic groups. However, in the latter context, the deformation tools become inappropriate.

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Jonas Deré KU Leuven Kulak

Anosov diffeomorphisms on infra-nilmanifolds

One of the first examples of a dynamical system with global hyperbolic behavior was given by Arnold's cat map, the diffeomorphism of the 2-torus T^2 which is induced by the matrix

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}.$$

Note that A only has eigenvalues of absolute value different from 1. This diffeomorphism has interesting dynamical properties, e.g. the periodic points are dense in T^2 and the map is structurally stable, meaning that every small distortion is topologically conjugate to the original map.

Roughly said, a diffeomorphism of a closed manifold is called Anosov if the tangent bundle splits into two df-invariant vector bundles E^s and E^u such that f contracts E^s and expands E^u. The example above is an Anosov diffeomorphism where the vector bundles are induced by the eigenspaces of the matrix A. Every Anosov diffeomorphism has similar properties as Arnold's cat map and thus they form an interesting type of dynamical systems to study.

All known examples of Anosov diffeomorphisms are of algebraic nature. For example, every Anosov diffeomorphism on an n-torus is topologically conjugate to a diffeomorphism induced by an integer matrix on the universal cover $\mathbb{R}^n$, just like the example above. It is conjectured that every Anosov diffeomorphism is topologically conjugate to a hyperbolic infranilautomorphism, where infra-nilmanifolds form the natural generalization of tori.

My research is focused on the classification of all compact manifolds (up to homeomorphism) which admit an Anosov diffeomorphism. Since every manifold with an Anosov diffeomorphism is conjectured to be homeomorphic to an infra-nilmanifold, we focus our attention on these manifolds. By studying a rational representation of the holonomy group, I was able to classify all infra-nilmanifolds modeled on a free nilpotent Lie group which admit an Anosov diffeomorphism. This is a generalization of a statement of Porteous (see [1] for more details). Currently, I am working on a classification of infra-nilmanifolds with Anosov diffeomorphisms in low dimensions and on some minimality questions concerning the dimensions of E^s and E^u.

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Groups acting on infinite dimensional non-positively curved spaces

Groups are usually better understood through their actions on nice spaces. Groups acting on non-positively curved spaces have been widely studied, in particular when spaces on which groups act are locally compact. If one removes the assumption on local compactness, results can become false but there are nice spaces which are not locally compact but which seem to be nice enough to let actions of certain groups be understood.

A large class of non-positively curved spaces is given by spaces associated to semi-simple algebraic groups over local fields. Those are symmetric spaces of non-compact type in the Lie case and Bruhat-Tits buildings in the non-Archimedean case. There exist analogs in infinite dimension that are infinite dimensional symmetric spaces of finite rank and non-discrete Euclidean buildings. Using differential geometric methods or ergodic theory one can prove rigidity theorems in the spirit of Mostow rigidity and Margulis superrigidity theorems for actions of lattices in semi-simple algebraic groups on these spaces.

Infinite dimensional spaces of non-positive curvature appear in some situations naturally. For example, the famous invariant subspace problem for operators on the separable Hilbert space can be rephrased using the geometry of an analog of the symmetric space $SL_n(\mathbb{R})/SO_n(\mathbb{R})$. Some actions on the circle can be studied using the universal Teichmuller space which has a component which is a Riemannian manifold of negative curvature and infinite dimension.

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Polynomially bounded cohomology for p-adic groups

Bounded cohomology was first introduced by Gromov [2] for smooth manifolds. Later Monod [4] studied bounded cohomology for groups with arbitrary coefficients. In his article he addresses many problems, notably the **Problem P** (see below) which I am most interested in.

Let G be a reductive p-adic group. The standard cohomology is well-known for such groups at least for irreducible admissible coefficient modules, see Casselman's famous unpublished notes [1]. In the almost simple case, Klingler [3] built natural cocycles using the Bruhat-Tits building associated to G. The later is a G-space X which captures most of the geometric information of G, e.g. X and G are quasi-isometric. But bounded cohomology remains largely unknown.

While investigating the bounded cohomology of these groups Monod suggested one should use a generalized concept of polynomially bounded cohomology. Our aim is to fully answer the following problem when the local field is $\mathbb{Q}_p$.

Problem P. *Let $G = \mathbb{G}(F)$ be a simple group of F-rank $r > 0$ over a local field F. Quasify Klingler's cocycles in order to obtain new classes in* *degree $r+1$ for cohomology with polynomial growth degree $r-1$ (in suitable module).*

The problem implicitly asks to define properly the G-modules involved and the cohomology with polynomial growth. Ogle [6] has results in this direction for countable groups. Quasification of cocycles was first perform by Monod and Shalom in [5].

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RESEARCH STATEMENT

I am in my second year of the Ph.D. program, studying developments in geometric group theory. In particular, Stallings' work with coverings of graphs [1] has been the inspiration behind very, very many papers — including fairly direct applications to pairs of finitely generated subgroups [2] and less obvious applications with special cube complexes [3].

In [1], Stallings builds a canonical completion of a graph to a cover in order to prove Marshall Hall's theorem that any finitely generated subgroup of a free group is a free factor in a subgroup of finite index. In [3], the authors constructs an analogue of this canonical completion, using special cube complexes rather than graphs, which yields a number of fun results. For instance, quasi-convex subgroups of fundamental groups of special cube complexes are separable. Another technique in [1] is to examine the core of a graph, which proves that if the intersection of two finitely generated subgroups has finite index in both, then it also has finite index in the join of the two. A similar technique is applied in [2], which addresses pairs of finitely generated subgroups of infinite index in free groups.

Certainly there will be even more analogues and extensions of the remarkable [1] to other properties of groups. I have yet to choose a research project.

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Matthew Durham University of Illinois at Chicago

Coarse geometry of Teichmüller spaces and mapping class groups

I am currently a 5th year graduate student at the University of Illinois at Chicago, under the supervision of Daniel Groves. My research is focused on understanding the coarse geometry of Teichmüller space — the space of hyperbolic metrics on a surface S up to isotopy, $\mathcal{T}(S)$ — with both the Teichmüller and Weil-Petersson metrics. I am also quite interested in the mapping class group — the group of orientation-preserving homeomorphisms of S up to isotopy to the identity, $\mathcal{MCG}(S)$ — which is closely related and shares strong coarse geometric features with $\mathcal{T}(S)$ with both metrics.

Using work of Rafi [8], my first paper [1] adapts the Masur-Minsky hierarchy machinery [6] to build a graph called the *augmented marking complex* which is $\mathcal{MCG}(S)$-equivariantly quasiisometric to $\mathcal{T}(S)$ with the Teichmüller metric. Very briefly, $\mathcal{T}(S)$ with the Teichmüller metric can be thought of as a hyperbolic space (the *uniformly thick part*) with many interconnected product regions (the *thin parts*) [7] glued on in a complicated fashion determined by the curve complex [5]. I use the Masur-Minsky marking complex [6] as a model for the thick part and build out the augmented marking complex as a model for the thin parts. The augmented marking complex was recently used by Eskin-Masur-Rafi [2] to prove the rank conjecture for $\mathcal{T}(S)$ with the Teichmüller metric.

In my second paper, I study the action of finite order subgroups of $\mathcal{MCG}(S)$ acting on $\mathcal{T}(S)$ with the Teichmüller metric. In his famous solution [4] to the Nielsen Realization Problem, Kerckhoff showed that any such subgroup $H \leq \mathcal{MCG}(S)$ fixes at least one point in Teichmüller space.

Denote the set of H-fixed points by $Fix(H)$. Given any point $X \in Fix(H)$, H preserves the metric on X and $Fix(H)$ is an isometrically embedded copy of the Teichmüller space of the quotient orbifold X/H inside of $\mathcal{T}(S)$. As a subspace of $\mathcal{T}(S)$, $Fix(H)$ is convex. Given the dearth of convex subsets of $\mathcal{T}(S)$ with the Teichmüller metric — other than such fixed point sets, only Teichmüller geodesics and special isometrically embedded copies of $\mathbb{H}^2$ called *Teichmüller disks* are known to be convex — one might ask whether $Fix(H)$ can be enlarged in a natural way to a larger convex or quasiconvex set.

For any constant $R > 0$, set $Fix_R(H) = \{X \in \mathcal{T}(S) | diam_{Teich}(H \cdot X) < R\}$ to be the R *almost-fixed* points of H. In a negatively curved space, one would expect $Fix_R(H)$ to be a convex regular neighborhood of $Fix(H)$, but given the product structure of the thin parts of $\mathcal{T}(S)$, it is *a priori* unclear that $Fix_R(H)$ is even connected, let alone convex or contained in a bounded neighborhood of $Fix(H)$.

My work answers some of these questions. First, I show that for any R, $Fix_R(H)$ is contain in a uniformly bounded regular neighborhood of $Fix(H)$. I also show that any point $X \in \mathcal{T}(S)$ has a fixed barycenter. Both of these results are easy in a nonpositively curved space, such as $\mathcal{T}(S)$ with the Weil-Petersson metric. By contrast, I build a large family of examples which show that $Fix_R(H)$ is generally not quasiconvex, which strongly uses the product structure of the thin parts of $\mathcal{T}(S)$.

I am also quite interested in hyperbolic 3-manifolds, CAT(0) cubical geometry, RAAGs, and $Out(F_n)$.

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Growth and other properties of branch groups

I have just finished my thesis in Oxford and am now a PostDoc at Paris-Sud. My research focuses on branch groups, in particular their growth. These are groups of automorphisms acting on a rooted tree, which does not necessarily have to be regular. I further studied word-relations in such branch groups and other rare algebraic properties they have.

My thesis evolved mostly around a group acting on a spherically homogenous tree, which is given by a sequence $\{l_i\}_{i\geq 0}$ of distinct primes, $l_i \geq 3$. A group acting on such a tree has a generator of infinite order and is virtually torsion free. However, it also falls into the class of branch groups and constitutes a first known example of a branch group that has exponential growth but does not contain any free subgroups as the following Theorems [1] imply:

Theorem A. *Let G be a certain branch group acting on a spherically homogenous rooted tree given by a sequence $\{l_i\}_{i\geq 0}$ of distinct odd primes. Then if l_i is such that $l_i \geq 36^i$ for each i, then there exists for every $g, h \in G$ a word $1 \neq w_{g,h}(x,y) \in F(x,y)$, where $F(x,y)$ is the free group on two generators, such that $w_{g,h}(g,h) = 1 \in G$.*

The proof of this Theorem gives an explicit construction of such a word, which only depends on the length of the chosen elements g and h.

Using the self-similarities within G, it is also possible to prove that G has exponential growth under certain assumptions:

Theorem B. *Let G be a group as above. Then G has exponential growth, if l_i is such that $l_i \geq C^{3^{i+1}(2+i)+1}$.*

Similar methods can be applied to obtain growth estimates for a wider class of branch and self-similar groups.

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Giovanni Gandini University of Copenhagen

Finiteness properties of groups

My research is mostly concerned with questions regarding *cohomological dimensions* and other *finiteness properties* of groups. At the moment I am thinking about the *Bridson-Kropholler conjecture*, which claims that every group of rational homological dimension one is a filtered colimit of groups of rational cohomological dimension one. I am also interested in the *Farrell-Jones conjecture*, and in *homological stability* for sequences of automorphism groups.

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Łukasz Garncarek Mathematical Institute, University of Wrocław

Irreducibility of quasiregular representations

Consider a group G acting on a measure space (X, μ), in such a way that the push-forward measure $\gamma_* \mu$ has a density ρ_γ with respect to μ, for all $\gamma \in G$. In such a setting we may define a series $\{\pi_s\}_{s\in\mathbb{R}}$ of unitary representations of G on the space $L^2(X, \mu)$ by the formula

$$\pi_s(\gamma)f = \rho_\gamma^{1/2+is} f \circ \gamma^{-1}.$$

I am interested in irreducibility of such representations, coming from some natural actions of groups on measure spaces. It is a mixing property stronger than ergodicity, and can be thought of as "quantum ergodicity". In my MSc thesis [1] I established it for the group of symplectomorphisms of a symplectic manifold, and the group of contactomorphisms of a contact manifold. In [2] I did it for Thompson's groups F and T, acting on the interval and the circle, respectively. Currently, I am looking into the case of the action of a δ-hyperbolic group on its boundary.

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Maxime Gheysens École Polytechnique Fédérale de Lausanne

Structure of CAT(0) spaces and of their isometry groups

I am mostly interested in the geometry of CAT(0) spaces and in group actions on these spaces, especially rigidity of the latter (i.e. fixed point properties). Several beautiful results are known to hold in the locally compact setting, for instance:

(1) equivariant splitting as a product of Euclidean spaces, symmetric spaces and spaces with totally disconnected isometry group (see [3, Theorem 1.1]);
(2) characterization of symmetric spaces and buildings in that setting (see [3, Theorem 1.3, Theorem 1.4]);
(3) superrigidity of actions of irreducible uniform lattices in product of groups (this holds without local compactness, see [2, Theorem 6]);
(4) characterization of amenable isometry groups of proper cocompact CAT(0) spaces (see [5, Theorem A]).

However, very few results are known in the much wilder realm of non locally compact CAT(0) spaces. Most of my research focuses on understanding these more pathological spaces.

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Special cubes complexes and ramified covering

I begin my second year of PHD and I'm interested in groups acting on $CAT(0)$ cube complexes. In my first year of PHD, I worked on fundamental groups of manifolds constructed by Gromov and Thurston in [1]. Theses constructions provide examples of manifolds of dimension $n \geq 4$ with sectional curvature as close to -1 as we want, but which do not admit a Riemannian metric with constant curvature. To do that, Gromov and Thurston consider some cyclic ramified cover of simple arithmetic manifolds. We have a lot of examples of groups of hyperbolic manifold which are virtually special, due to Agol [2] and Wise, all 3-manifold groups are virtually special. Other examples are the simple arithmetic manifolds. It's a result of Bergeron, Haglund and Wise [3]. So I prove, using simple ramified coverings of cube complexes, that Gromov and Thurston's examples are virtually special too. I am curently trying to generalise the notion of ramified coverings of cube complexes, and find conditions to keep the special property, in order to find new special groups.

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Graphical small cancellation groups

My research is focused on (infinitely presented) graphical small cancellation groups. Graphical small cancellation theory is a generalization of classical small cancellation theory that allows constructions of groups with prescribed embedded subgraphs in their Cayley graphs. It was introduced by Gromov, extending ideas of Rips and Segev.

Using a geometric version of this new theory, Gromov constructed *Gromov's monster*, a lacunary hyperbolic group that coarsely contains an expander graph [1, 2]. This group is currently the only known group that does not coarsely embed into a Hilbert space, and it is a promising candidate for a counterexample to the Baum-Connes conjecture. (It already provides a counterexample to a stronger variant of the conjecture, the Baum-Connes conjecture with coefficients [4].)

I am interested in the combinatorial interpretation of graphical small cancellation theory. In this context, the graphical generalizations of the classical non-metric $C(n)$ small cancellation conditions are natural. I will briefly give definitions and first results.

Let S be a finite set. A *labelled graph* Γ over the alphabet S is a graph each edge of which is assigned an orientation and an element of S. The *group defined by* Γ, denoted $G(\Gamma)$, is given by the presentation $G(\Gamma) := \langle S \mid \text{words read on closed paths on } \Gamma \rangle$.

A *piece* on Γ is a labelled path that can be read from two distinct vertices of Γ. The labelled graph Γ satisfies the *graphical $C(n)$ condition* if no nontrivial closed path on Γ is the concatenation of fewer than n pieces and if the labels of reduced paths are freely reduced words.

In my paper [3], I established basic properties of graphical $C(6)$ and $C(7)$ groups. Besides generalizations of fundamental facts about classical $C(6)$ and $C(7)$ groups, the following are main results:

Theorem. Let Γ be a $C(7)$-labelled graph. Then $G(\Gamma)$ contains a non-abelian free subgroup unless it is trivial or infinite cyclic.

Theorem. Let $(\Gamma_n)_{n\in\mathbb{N}}$ be a sequence of finite, connected graphs such that their disjoint union Γ is $C(6)$-labelled. Then the coarse union of the Γ_n coarsely and injectively embed into $\mathrm{Cay}(G(\Gamma), S)$. If, moreover, Γ is $C(7)$-labelled, then there exists an infinite subsequence of graphs $(\Gamma_{k_n})_{n\in\mathbb{N}}$ such that $G(\sqcup_{n\in\mathbb{N}}\Gamma_{k_n})$ is lacunary hyperbolic.

Thus the $C(7)$ condition permits constructions of non-amenable, lacunary hyperbolic groups that coarsely contain prescribed infinite sequences of finite graphs.

Most existing constructions of graphical small cancellation groups with extreme properties use geometric small cancellation conditions and rely on probabilistic arguments, whence they are non-explicit. One of my main objectives is to construct such groups explicity using the more easily accessible graphical $C(n)$ conditions. In this context, expander graphs in particular have drawn my interest.

Another main objective is to establish further group-theoretic and analytic properties of (infinitely presented) graphical small cancellation groups to better understand this class of groups.

My research is supervised by my advisor Goulnara Arzhantseva and supported by her ERC grant "ANALYTIC" no. 259527.

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Thomas Haettel Université Montpellier 2

Asymptotic geometry of homogeneous spaces and nonpositive curvature of braid groups

I am interested in understanding the asymptotic geometry of homogeneous spaces, such as the space of maximal flats of a symmetric space of non-compact type or of a Euclidean building ([1],[2]).

I am also interested in CAT(0) spaces and groups. With Dawid Kielak and Petra Schwer, we have been studying simplical $K(\pi, 1)$'s for braid groups and other Artin groups of finite type, described by Tom Brady and Jon McCammond ([3],[4]). We have proved that some of them are CAT(0) ([5]).

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Tobias Hartnick Technion, Haifa

Cohomological boundary rigidity

At the moment I am interested in the following problem: Let G be a topological group, $H_c^\bullet(G;\mathbb{R})$ its continuous group cohomology and $H_{cb}^\bullet(G;\mathbb{R})$ its continuous bounded group cohomology. What can you say about the natural comparison map

$$H_{cb}^\bullet(G;\mathbb{R}) \longrightarrow H_c^\bullet(G;\mathbb{R})?$$

For a discrete group G, the right hand side captures topological information about the classifying space BG, while the left hand side captures information about the possible outcomes of random walks on the group. Thus the comparison map relates asymptotic probabilistic information to topological information. It is far from understood even in the simplest non-amenable cases, such as for free groups. In this specific case, one can try to use the fact that a free group is a lattice in $\mathrm{SL}_2(\mathbb{R})$ and coamenable to a lattice in $\mathrm{SL}_2(\mathbb{C})$

in order to obtain cohomological information. Thus, even if one is originally only interested in discrete groups, one is naturally led to study the comparison map for Lie groups. In this case, techniques from analysis (partial differential equations, harmonic analysis, integral transforms) are available. This machinery has only very recently been set up properly (see [1]), and we are now curious to see what our Lie theoretic computations imply for the study of discrete groups.

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Jesús Hernández Hernández Université Aix-Marseille

Rigidity of the mapping class group and associated structures

I have just started my PhD supervised by Dr. Hamish Short and Dr. Javier Aramayona at the Aix-Marseille Université. My research focuses on the rigidity phenomena of the mapping class group and combinatorial structures associated to compact surfaces.

The parallels between the behaviour of the mapping class group of a compact surface of negative Euler characteristic and that of arithmetic/algebraic groups and arithmetic lattices, have raised many analogies between properties of one and the other. In particular, the rigidity (and superrigidity) problem of the mapping class group is related to the rigidity result in semisimple Lie groups known as Mostow rigidty (Margulis superrigidity). While there have been several advances pertaining the rigidity problem by N.V. Ivanov, J. Behrstock, D. Margalit, E. Irmak, K.J. Shackleton, among others, I will be focusing on extending the rigidity results of several combinatorial structures, such as the Curve complex and the Hatcher-Thurston complex, into superrigidity results; this will have the benefit of not only being applied to the rigidity of the mapping class group, but also to that of other structures associated to a compact surface such as the Teichmüller space and the Moduli space, among others.

Among the techniques used here, are those developed on the study of large-scale geometry and hyperbolic (and relatively hyperbolic) groups. The reason for this is the need of the study of different kinds of elements of the mapping class group and the study of the different properties needed in the homomorphisms between (a priori) different mapping class groups to achieve the rigidity.

Given that I have just started my PhD not too long ago, I have no results to present here as my own yet.

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Camille Horbez Université de Rennes 1

Horoboundary of outer space, random walks, and Out(F_N)-complexes

My research concentrates on the study of the group $\mathrm{Out}(F_n)$ of outer automorphisms of a finitely generated free group via its action on several geometric complexes, such as Culler and Vogtmann's outer space CV_n, or complexes of splittings of F_n.

I have been studying a compactification of outer space via horofunctions for the asymmetric metric. This was motivated by a desire to understand the behaviour of random walks on $\mathrm{Out}(F_N)$: a theorem of Karlsson and Ledrappier provides conditions under which a random walk on a group acting on a metric space follows a random direction directed by some horofunction. This can be used to derive an Oseledets-like theorem for random automorphisms of free groups.

Describing the horocompactification of outer space turns out to be related to the question of spectral rigidity of the set $\mathcal{P}_N$ of primitive elements of F_N in F_N-trees: what can we say of two trees $T, T' \in \overline{cv_N}$ whose translation length functions are equal in restriction to $\mathcal{P}_N$? I have constructed a class of examples of trees having this property, and shown that this class of examples provides the only obstruction to spectral rigidity of the set of primitive elements of F_N in $\overline{cv_N}$.

I am more generally interested in developing tools for studying random walks on $\mathrm{Out}(F_N)$, in particular adapting Kaimanovich and Masur's techniques for mapping class groups to the $\mathrm{Out}(F_N)$ case. For instance, one might ask conditions under which a random walk on outer space converges to some boundary point, or ask for generic properties of elements of $\mathrm{Out}(F_N)$ obtained by following a sample trajectory during sufficiently long time. Random walks can be used for instance to study subgroups of $\mathrm{Out}(F_N)$, or to describe the Poisson boundary of $\mathrm{Out}(F_N)$.

Finally, I have some interest in investigating the geometry of some hyperbolic $\mathrm{Out}(F_N)$-complexes, among which stand the complexes of free or cyclic splittings of F_N. In a joint work with A. Hilion, we gave a new proof of the hyperbolicity of the free splitting complex $\mathcal{FS}_N$, due to Handel and Mosher, by understanding the geometry of surgery paths in a dual model of $\mathcal{FS}_n$ defined in terms of spheres. I am now interested in questions such as describing the Gromov boundaries of these complexes.

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David Hume Université Catholique de Louvain

Research statement

Coarse embeddings of discrete metric spaces into Banach spaces have been an important topic in computer science for many years and more recently became so in geometric group theory, combinatorics and K-theory. At the suggestion of Gromov, Yu and later Kasparov and Yu proved that any group admitting a coarse embedding into any uniformly convex Banach space satisfies the Novikov and coarse Baum-Connes conjectures. However, Gromov proved that there exist finitely generated groups which do not admit a coarse embedding into any Hilbert space.

A stronger notion was introduced by Guentner and Kaminker, called the compression exponent, $\alpha^*_Y(X)$. This is defined as the supremum over $\alpha \in [0,1]$ such that there exists a map $\phi : X \to Y$ such that, for some $C \geq 1$,

$$C^{-1}d_X(x_1,x_2)^\alpha - C \leq d_Y(\phi(x_1),\phi(x_1)) \leq Cd_X(x_1,x_2+C$$

We denote by $\alpha^*_p(X)$ the value $\alpha^*_{\ell^p(\mathbb{N})}(X)$, and call this value the ℓ^p compression exponent.

My previous work has been to calculate the compression exponent of various important families of groups (cf. [Hum11, Hum12]).

Theorem 1 *Let G be a finitely generated group which is hyperbolic relative to a collection of subgroups $\{H_i \mid i = 1,\ldots,n\}$. For all $p \geq 1$, $\alpha^*_p(G) = \min\{\alpha^*_p(H_i) \mid i = 1,\ldots,n\}$.*

Theorem 2 *Let S be a compact oriented surface. Then $\alpha^*_p(\mathrm{MCG}(S)) = 1$ for every $p \geq 1$.*

The same questions can be asked for equivariant embeddings, that is embeddings $\phi : G \to X$ such that for some action of G on X, $(g, x) \mapsto g \cdot x$, ϕ is the orbit map of the action with respect to some point $e \in X$, i.e. $\phi(g) = g \cdot e$ for every $g \in G$. The equivariant ℓ^p compression exponent $\alpha_p^{\#}(G)$ is then given by the same supremum as $\alpha_p^*(G)$ but taken over only equivariant embeddings. Unlike the situation for $\alpha_p^*(G)$ very few real numbers are known to be the equivariant compression exponent of a finitely generated group. This is one of my current research interests.

My other current interests include the geometry of (permutation) wreath products, spaces with no convex splittings, and constructions of relatively hyperbolic groups with given peripheral subgroups.

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Andrew Kelley Binghamton University

Research statement

Earlier in 2013, I started studying the Banach-Tarski paradox and amenability from Stan Wagon's book [1]. I then spent the summer studying the paper by Kate Juschenko and Nicolas Monod [2], demonstrating the amenability of a known uncountable collection of finitely generated simple groups (which are infinite). Each group arises from a homeomorphism T of the Cantor set, which is minimal, that is, that for every point p in the Cantor set, $\{T^n(p) | n \in \mathbb{Z}\}$ is dense, or equivalently, that there is not proper subset of the Cantor set, invariant under T.

After this paper, I was planning on reading the newer paper [3], which "gives a more straightforward and conceptual proof of [the previous paper]." In this paper, amenability of a larger class of groups is connected to whether or not a certain natural random walk is recurrent.

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Dawid Kielak RFW-Universität Bonn

Recent activity

Out(F_n) and Out(RAAG)

I am interested in the outer automorphism groups of free groups. In particular I would like to understand their linear representations and rigidity of homomorphisms between different groups in the family. I am also very much interested in property (T) and property FAb for this class of groups.

More recently I started thinking (together with Sebastian Hensel and Piotr Przytycki) about the Nielsen realisation theorem for Out(RAAG). Fascinating as it is in its own right, it is also an important tool that should allow a lot of techniques from Out(F_n) to be adapted to this more general collection of groups.

Curvature of braid groups

I am also investigating simplicial $K(\pi, 1)$'s for braid groups (defined by Brady–McCammond). Together with Thomas Haettel and Petra Schwer we managed to prove that this space associated to the 6-strand braid group is CAT(0).

Left-invariant orderings and fractions

Another area of my recent activity is the theory of left-invariant orderings, and associated problems in the structure of semigroups of groups. In particular I am interested in geometric ideas that would help answering questions related to the structure of spaces of left-orderings for groups, which can be understood via their geometry (e.g. (relatively) hyperbolic groups, groups with infinitely many ends).

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Juhani Koivisto University of Helsinki,

Department of Mathematics and Statistics

Analysis in metric spaces

My area of research is curvature in metric spaces and rigidity properties of groups. In particular, I have looked into generalizations of Kazhdan's Property (T) in the setting of reflexive Banach spaces obtaining a vanishing condition for 1-cohomology with coefficients in uniformly bounded representations, [1]. Currently, I have been looking for a similar vanishing condition for isometric representations, and as an intermediate step, at the higher cohomology groups with coefficients in uniformly bounded representations. In the metric space setting, I have been looking at possible obstructions to weighted Poincaré inequalities using the controlled coarse homology of P. W. Nowak and J. Špakula describing "how amenable" a space is [2].

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Christian Lange Universität zu Köln

The underlying space of an orbifold

I am a first-year PhD student of Alexander Lytchak in Cologne. I started my PhD project by answering the question posed by Davis: "When is the underlying space of an orbifold a topological manifold?" [1]. It amounts to the classification of finite groups acting isometrically on a Euclidean vector space such that the quotient space is homeomorphic to the original vector space. The solution to this problem requires techniques from geometry, topology and group theory. I obtained the following result [2].

Theorem. For a finite subgroup $\Gamma < O(n)$ the quotient space $\mathbb{R}^n/\Gamma$ is homeomorphic to $\mathbb{R}^n$ if and only if Γ has the form

$$\Gamma = \Gamma_{ps} \times P_1 \times \ldots \times P_k$$

for a pseudoreflection group Γ_{ps} and binary icosahedral groups $P_i < SO(4)$, $i = 1, \ldots, k$, such that the factors act in pairwise orthogonal spaces and such that $n > 4$ if $k = 1$.

Pseudoreflection groups are finite linear groups generated by rotations, i.e. by orthogonal transformations with codimension two fixed point subspaces. Examples are orientation preserving subgroups of real reflection groups and unitary reflection groups considered as real groups.

Currently, I am working on extensions of my result to other categories and on the analogous question for manifolds with boundary. A related problem is a question by Gromov, if there are compact manifolds in arbitrary dimensions that can be written as a quotient of a hyperbolic space by an isometric and discrete group action.

In general, I am interested in connections between geometry, topology and group actions and I would like to learn more about geometric group theory, in particular.

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Nir Lazarovich Technion, Haifa

Regular CAT(0) polygonal/cubical complexes

My main research interest is group actions on polygonal and cube complexes.

Given a natural number $k \geq 3$ and a finite graph L (respectively a finite flag simplicial complex L), it is natural to consider the CAT(0) (k, L)-complexes (resp. CAT(0) L-cube-complexes); these are polygonal complexes (resp. cube complexes) obtained by gluing regular k-gons (resp. cubes) such that at each vertex the link is isomorphic to L. The study of these complexes may provide various examples for geometric group actions which exhibit interesting algebraic and geometric properties.

A natural question arises: can one give a necessary and sufficient condition on the pair (k, L) (resp. on the complex L) such that there is a unique, up to isomorphism, CAT(0) (k, L)-complex (resp. L-cube-complex)? The few known examples of unique (k, L)-complexes have provided a fertile ground for many theorems.

Thus far, we were able to answer this question fully for the pair (k, L) when k is even and for cube complexes. The main result describes a simple combinatorial condition, called superstar-transitivity, on L for which there exists at most one (k, L)-complex (resp. L-cube-complex). This condition is also sufficient for uniqueness in pairs (k, L) where k is odd.

In light of these results, it is clear that complexes with superstar-transitive links play a special role in the world of polygonal/cube complexes. The aim of this research is to investigate the special properties of these complexes through the study of the general theory.

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Adrien Le Boudec Université Paris-Sud 11 (Orsay)

Large scale geometry of groups

My research has been so far mainly concerned with the asymptotic geometry of discrete and locally compact groups.

Asymptotic cones. An asymptotic cone of a locally compact group G endowed with some word metric is a metric space which is a sort of picture of G seen from infinitely far away. The construction of asymptotic cones depends on the choice of a scaling sequence and a non-principal ultrafilter, and therefore a group G may have different asymptotic cones, not containing the same information about large scale geometric properties of G.

One of the main question I am focusing on is the study of the existence of cut-points in asymptotic cones. Recall that a point in a connected metric space is a cut-point if we get a disconnected space when we remove it. Having cut-points in some asymptotic cones is closely related (Drutu-Mozes-Sapir) to a geometric property of the group G called divergence, which, roughly, estimates how hard it is to connect points in G while avoiding a large ball.

Almost automorphisms of trees. Recently I have also been interested in the group $\mathrm{AAut}(T)$ of almost automorphisms of a locally finite tree T. This group does not act on the tree but on its boundary ∂T. Roughly speaking, an almost automorphism of T is a homeomorphism of ∂T which is piecewise a tree automorphism. This group carries a natural locally compact and totally disconnected topology.

In the case of the non-rooted regular tree $\mathcal{T}_d$, the almost automorphism group $\mathrm{AAut}(\mathcal{T}_d)$ was introduced by Neretin and later Kapoudjan proved that $\mathrm{AAut}(\mathcal{T}_d)$ is a simple group. Recently, the work of Bader, Caprace, Gelander and Mozes shed light on some very interesting property of this group: although locally compact simple groups usually tend to have lattices, they proved that $\mathrm{AAut}(\mathcal{T}_d)$ does not follow this rule. Actually $\mathrm{AAut}(\mathcal{T}_d)$ turned out to be the first example of a locally compact simple group without lattices.

When T is a quasi-regular tree, the almost automorphism group of T contains R. Thompson's groups and their generalizations usually called Higman-Thompson's groups. One of the reasons why people became interested in these finitely generated groups is because of the combination of simplicity and finiteness properties. While simplicity results for $\mathrm{AAut}(T)$ have already been studied, I recently proved that the group $\mathrm{AAut}(T)$ is compactly presented. Additionally, I also obtain an upper bound on its Dehn function in terms of the Dehn function of the embedded Higman-Thompson's group.

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Nils Leder Universität Münster

Research statement

I am a second year Master student at the WWU Münster, Germany, but spend the winter semester 2013/14 as an ERASMUS student at Paris 13. My interest for (geometric) group theory arose from lectures by Prof. Linus Kramer and by Prof. Arthur Bartels on this topic.
I wrote my bachelor thesis on “The Complex of Free Factors of a Free Group” related to the equally named article of Allen Hatcher and Karen Vogtmann.

I am also interested in the structure of group rings (in particular Kaplansky’s conjecture that for each torsion-free group G the group ring $\mathbb{C}[G]$ contains only the trivial idempotents 0 and 1) and connections between group theory and algebraic topology as graphs of groups.

My master thesis is planned to be about homological stability for Coxeter groups.

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Robin Loose Universität Münster

Statement of interests

This year is my second year as a master student at the WWU Münster. So far I have attended several courses in Group Theory, Geometry and Topology in Münster and Barcelona, where I studied for half a year as an Erasmus student last year. For me one of the most fascinating features of modern mathematics is exchanging methods and techniques of different areas. On the one hand to develop new theories and raise interesting questions, on the other hand in order to solve pure (algebraic/geometric/topological) problems. At the moment I am studying Spaces of Non-positive Curvature, Characteristic Classes, Index theory, L^2-Invariants and Algebraic K-Theory.

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John Mackay University of Bristol

Analysis on boundaries of hyperbolic groups

I am interested in using tools from analysis to study hyperbolic and relatively hyperbolic groups.

The boundary at infinity of a hyperbolic group is a canonically defined topological space, and there are many interesting links between the topological properties of the boundary and algebraic properties of the group. For example, local cut points in the boundary correspond to splittings of the group over virtually cyclic subgroups [Bowditch].

However, the boundary has a metric structure too, which is canonical up to 'quasisymmetric' homeomorphisms (these are kind of like conformal or quasi-conformal homeomorphisms in the plane). Pansu's conformal dimension is a variation on Hausdorff dimension which is relevant for these spaces, and gives a quasi-isometric invariant of the hyperbolic group.

I'm interested in using conformal dimension to study random groups (which typically are hyperbolic). Other tools from analysis like quasi-circles are useful in this context too. I'm also working with Alessandro Sisto on extending some of these applications from hyperbolic to relatively hyperbolic groups.

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Alba Marina Málaga Sabogal Université Paris-Sud 11 (Orsay)

Some properties of a family of transformations on a discrete cylinder

I work on a family of (discrete) dynamical systems which is heuristically related to a billiard on a parallelogram. This family is defined on the discrete cylinder $\mathbb{S}^1 \times \mathbb{Z}$ where $\mathbb{T}^1 = \mathbb{R}/\mathbb{Z}$ is the one-dimensional torus (i.e. the circle). For any bi-infinite sequence $\underline{\alpha} \in \mathbb{T}^{\mathbb{Z}}$, we define the transformation $F_{\underline{\alpha}}$ almost everywhere on the cylinder as follows:

$$F_{\underline{\alpha}}([x]_{\mathbb{Z}}, n) = \left([x+\alpha_n]_{\mathbb{Z}}, n + \left\{ \begin{array}{ccc} 1 & if & x+\alpha_n \in (0, \frac{1}{2}) + \mathbb{Z} \\ -1 & if & x+\alpha_n \in (-\frac{1}{2}, 0) + \mathbb{Z} \end{array} \right. \right).$$

When the sequence $\underline{\alpha}$ is constant and irrational, Conze and Keane showed in [1] that $F_{\underline{\alpha}}$ is ergodic.

I am trying to understand what are the typical properties of $F_{\underline{\alpha}}$ in the following meaning. Namely, what properties hold for almost any $\underline{\alpha}$ or for a generic $\underline{\alpha}$ in the parameter space? For the moment I have proved that conservativity is both generic and almost-sure, whereas the minimality is generic. I would like to understand also the diffusion properties of this family.

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Michał Marcinkowski Wrocław

My research interests include:

(1) **Biinvariant word length**. Let G be a group generated by a symmetric set S and let $\overline{S}$ be the minimal conjugacy invariant set containing S. The biinvariant word metric, denoted $||.||$, is the word metric defined with respect to the (in most cases infinite) set $\overline{S}$. It may be dramatically different from the standard word metric (e.g. $SL(n,\mathbf{Z})$ is bounded in $||.||$). I am interested in the geometry of groups equipped with the biinvariant metric, especially in metric behavior of cyclic subgroups (i.e. distorsion).

(2) **Coarse homology and macroscopic dimension**. Recently coarse homology theories were used to give homological characterisations of geometrical notions such as macroscopic dimension (A.Dranishnikov) and topological amenability (J. Brodzki, P.W. Nowak, G.A. Niblo, N. Wright). I am interested in possible further applications of coarse homologies in these directions.

(3) **a-T-m property**. We say that a group is a-T-m(enable) if it admits a proper affine action on a Hilbert space. Every such action consists of a linear transformation and a 1-cocycle. I am interested in methods of constructing affine representations as well as in growths of 1-cocycles.

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Marco Marschler Bielefeld University

Classical finiteness properties of certain families of groups

My research focuses on *classical finiteness properties* of groups. Recall that a group G is said to be of *type* F_n if it admits an Eilenberg–MacLane space with finite n-skeleton and it is of *type* F_∞ if it is of type F_n for all n.

The class of groups I am mainly interested in are *parabolic subgroups* of S-arithmetic subgroups of reductive algebraic groups over function fields. My PhD-project, under supervision of Prof. Kai-Uwe Bux, is to determine the finiteness properties of such groups. Another project, joint with Bux, Martin Fluch, Stefan Witzel and Matthew Zaremsky, was concerned with the finiteness properties of some *generalizations of Thompson's groups*, such as the braided versions V_{br} and F_{br} of V and F and the higher-dimensional Brin–Thompson groups $sV, s \in \mathbb{N}$. Both projects include the proof of these groups being of type F_∞.

Parabolic subgroups: A subgroup $\mathcal{P}$ of a Chevalley group $\mathcal{G}$ is called *parabolic* if it contains a Borel subgroup $\mathcal{B}$. Let S denote a finite set of places of the global function field K and let $\mathcal{O}_S$ be the ring of S-integers. By work of Bux, Ralf Köhl and Witzel, $\mathcal{G}(\mathcal{O}_S)$ is known to be of type F_{d-1} but not of type F_d, where d equals the sum over the local ranks of $\mathcal{G}$ over the completions $K_p, p \in S$. On the other hand, Bux proved in his thesis, that the groups $\mathcal{B}(\mathcal{O}_S)$ are of type $\mathrm{F}_{|S|-1}$ but not of type $\mathrm{F}_{|S|}$. In particular the finiteness properties of the Borel subgroups only depend on the number of places, not on the local ranks. It is therefore a natural thing to ask for the finiteness properties of proper parabolic subgroups in between $\mathcal{B}$ and $\mathcal{G}$ and the behaviour of the finiteness length as the groups $\mathcal{P}$ grow. Both of the above results were obtained by considering actions of the groups on *affine buildings* and using tools like *discrete Morse theory* to analyze these actions. It stands to reason that the same tools can be used to determine the finiteness properties of the parabolic subgroups.

Thompson's groups: Elements of Thompson's group F can be heuristically described in terms of "strands". A single strand can split into two and two adjacent strands can merge into one. This point of view leads to the well-known description of F by paired tree diagrams or "split-merge" diagrams. If in addition the strands in these diagrams are allowed to braid with each other, one obtains a description of the *braided Thompson group* V_{br} as intoduced by Brin and shown to be finitely presented. In joint work with Bux, Fluch, Witzel and Zaremsky we recently proved that V_{br} is even of type F_∞. This was done by considering the action of V_{br} on a space X that is termed the *Stein space* for V_{br}. This space is a retract of the "natural" V_{br}-space, that is the realization of the poset of the split-braid-merge diagrams. The retraction to the Stein space proved to be one of the key steps in proving F_∞. In particular the descending links in X are closely related to the well-studied matching complexes of graphs and lead to the notion of *matching complexes of arcs on surfaces*. The same idea of passage to a *Stein space* also allowed us to proof F_∞ for the pure braided Thompson group F_{br} and another generalization of V, namely the higher dimensional Brin–Thompson groups sV.

In recent years Thompson's groups have been generalized in various ways and directions, so that there are a lot more "Thompson–like" groups in the literature. For example Belk and Forrest introduced a *Thompson group for the Basilica* in 2012. This group T_B can be viewed as a generalization of Thompson's group T but instead of acting by homeomorphisms on the unit circle, it acts on the Julia set known as the Basilica. Belk and Forrest have shown this group to be finitely generated, that is T_B is of type F_1. But it is conjectured by Belk to be not finitely presented, i.e. not of type F_2. This would show, that there are Thompson–like groups that are not of type F_∞, which in turn renders the question of the finiteness length of generalized Thompson groups even more interesting.

It stands to reason that one could learn about the finiteness properties themselves by understanding their behaviour with respect to the different methods of generalization.

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Alexandre Martin Fakultät für Mathematik, University of Vienna

Combination problems in geometric group theory

I am currently interested in the following combination problem: given a group G acting cocompactly on a simplicial complex X, what can we say about G out of the geometry of X, the dynamics of the action, and the various inclusions of stabilisers of simplices?

I proved a combination result for hyperbolic groups [1] which generalises the acylindrical version of the Bestvina–Feighn theorem for graphs of groups to complexes of groups of arbitrary dimension.

Theorem. Let G be a group acting cocompactly without inversion by simplicial isomorphisms on a simplicial complex X. Suppose that:

- all the stabilisers of simplices are hyperbolic, and all the inclusions of stabilisers are quasiconvex embeddings,
- the complex is CAT(0) and hyperbolic,
- the action of G on X is acylindrical.

Then G is hyperbolic and the stabilisers of simplices embed in G as quasiconvex subgroups.

I am currently trying to drop the CAT(0) assumption to allow actions on complexes with a more combinatorial geometry, such as systolic complexes (joint project with D. Osajda) and quotient of trees under the action of a very rotating family.

Such cocompact but non-proper actions arise naturally in geometric group theory. Two examples I am particularly interested in are the following:

- given a group admitting a codimension one subgroup, the action on the associated CAT(0) cube complex;

- the action of a small cancellation group over a graph of groups on some appropriate 2-complex.

I am also studying other types of properties, such as the existence of a cubulation and residual finiteness.

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Measuring finiteness in groups

Growth functions in groups have been widely studied and provide tools to discriminate groups. In his celebrated 1981 theorem, Gromov showed how the *word growth* function (which counts the number of elements in the n-th ball of a finitely generated group) describes completely finitely generated virtually nilpotent groups as those with growth function bounded above by a polynomial. A similar result has been obtained in 1993 by Lubotzky, Mann and Segal (see [7]) which classified residually finite virtually solvable groups of finite rank using the *subgroup growth* function (which counts all finite index subgroups of at most at most a given index).

Recently Bou-Rabee [3] has introduced the *residual finiteness growth* function for finitely generated residually groups which attempts to quantify the residual finiteness of a group in an efficient way. More precisely, for a residually finite finitely generated group $\Gamma = \langle X \rangle$, one can define the *residual finiteness growth function* $F_{\Gamma,X}(n)$ as the minimal natural number N such that any element of word length $\leq n$ with respect to X can be detected in a quotient Q of cardinality $\leq N$. It can be shown that the function $F_{\Gamma,X}(n)$ is essentially independent of the generating set X. This function has been studied for several groups and it has been shown by Bou-Rabee and McReynolds [4] that, for the case of linear groups, it gives a Gromov-like characterization for virtually nilpotent groups. In joint work with Kassabov [5] we worked on the case of the free group by rephrasing the study of the growth in terms of laws in groups. Morever, we established a connection of the residual finiteness growth to a second growth called *intersection growth* which allows one to obtain information about the residual finiteness growth.

The intersection growth function $i_\Gamma(n)$ is defined to be the index of the the intersection of all subgroups of index at most n inside the finitely generated group Γ and can be seen as a variant of the subgroup growth function, but in many cases it is better behaved. In a current joint work with Biringer, Bou-Rabee and Kassabov [1, 2] we have estimates for the intersection growths for free groups, lamplighter groups, and finitely generated torsion-free nilpotent groups some arithmetic groups. Moreover, in the case of finitely generated torsion-free nilpotent groups, we are in the course finding parallels to Grunewald, Lubotzky, Segal and Smith's work on the relation between the subgroup growth and the nilpotency of a group (see [7]). Finally we observe that intersection growth can behave badly by constructing a residually finite group Γ where $i_\Gamma(n)$ is super linear on infinitely many numbers and which is below any strictly increasing function for infinitely many others (see the joint work with Kassabov [6])

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Measured flat laminations.

Let Σ be a connected orientable compact surface and m a hyperbolic metric on Σ. A *hyperbolic lamination* on (Σ, m) is a (non empty) closed subset of Σ which is disjoint union of simple local geodesics. A *measured hyperbolic lamination* is a hyperbolic lamination endowed with a tranverse measure, which is a family of Radon measures on transverse arcs to the lamination, whose support is their intersection with the lamination, and that have certain invariance properties (see for example [1]).

A *flat structure* on Σ is a Euclidean metric with conical singularities of angles $k\pi$, with $k \geqslant 3$, such that the holonomy of any piecewise $\mathcal{C}^1$ loop is contained in $\{\pm \mathrm{Id}\}$. We will denote its distance by d. In a series of two papers ([3] and [4]), I proposed an analog of measured hyperbolic laminations on surfaces endowed with a flat structure, that I have called *measured flat* *laminations*. The main goal is then to study the degeneration of flat structures on Σ, pursuing the approach of [2], but in terms of geometric objects instead of geodesic currents on the boundary at infinity.

Let $p : (\widetilde{\Sigma}, \widetilde{d}) \to (\Sigma, d)$ be a locally isometric universal covering. We denote by $\partial_\infty \widetilde{\Sigma}$ the boundary at infinity of the CAT(0) space $(\widetilde{\Sigma}, \widetilde{d})$. Two local geodesics of (Σ, d) are said to be *interlaced* if they have some preimages whose pair of endpoints separate each other in $\partial_\infty \widetilde{\Sigma}$. We endow the set of oriented local geodesics (defined up to reparametrization) with the quotient topology of the compact-open topology, by the action of $\mathbf{R}$ by translations.

Definition. A flat lamination on (Σ, d) is a (non empty) set Λ of oriented local geodesics (defined up to reparametrization), whose elements are called leaves, such that:

- leaves are not self-interlaced and two by two not interlaced;
- Λ is invariant by reversing the orientation of leaves;
- Λ is closed.

We will call *support* of Λ the union of the image of its leaves.

The main difficulties compared with hyperbolic laminations come from the fact that leaves are not necessarily disjoint and that the flat laminations are not determined by their support. It forces to define transverse measures to a flat lamination as a family of measures on local geodesics transverse to the transverse arcs to the lamination instead of measures on the transverse arcs.

The main result of these papers is a classification theorem of flat laminations on a compact surface endowed with a flat structure. I have also showed that every finite metric graph, except four, is the support of a geodesic lamination with uncountably many leaves, none of whose is eventually periodic.

We can define a natural (but non bijective) correspondence between measured flat laminations and measured hyperbolic laminations, that shows, among other properties, that the projective space of measured flat laminations is compact and that the images of measured flat laminations that are the union of finitely many periodic local geodesics, are dense in it. Moreover, we can define a dual tree to a measured flat lamination, and we show that it is isometric to the dual tree of the corresponding measured hyperbolic lamination.

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Flat surfaces and dynamics in moduli space

I am interested in the theory of translation surfaces and related problems in Teichmüller theory. By a *translation surface* we mean a pair (X, ω) where X is a compact Riemann surface (complex curve) and ω is a holomorphic one-form on X. It is well known that such a pair defines an Euclidean metric with conical singularities, and trivial linear holonomy on the underlying surface of X. For any fixed positive integer $g \geq 2$, the space of translation surfaces of genus g is the vector bundle $\Omega\mathfrak{M}_g$ over the moduli space $\mathfrak{M}_g$ of Riemann surfaces of genus g, the fiber over a point $X \in \mathfrak{M}_g$ is the (complex) vector space $\Omega(X)$ of holomorphic one-forms on X. Note that $\dim_{\mathbb{C}} \Omega(X) = g$.

The space $\Omega\mathfrak{M}_g$ is naturally stratified by the orders of the zeros of the one-form. Namely, for any integer vector $\underline{k} = (k_1, \ldots, k_n)$ with $k_i > 0$, we denote by $\Omega\mathfrak{M}_g(\underline{k})$ the set of pairs (X, ω) where ω has exactly n zeros with degrees given by $\underline{k}$. The space $\Omega\mathfrak{M}_g(\underline{k})$ is called a *stratum.* It is well known that $\Omega\mathfrak{M}_g(\underline{k})$ is a complex algebraic orbifold of dimension $2g+n-1$ which admits a complex affine structure, and carries a natural volume form μ. Moreover, there exists an action of $\mathrm{GL}^+(2, \mathbb{R})$ on $\Omega\mathfrak{M}_g(\underline{k})$, which preserves the complex affine structure, such that the volume form μ is preserved by $\mathrm{SL}(2, \mathbb{R})$.

It turns out that the understanding of this action of $\mathrm{GL}^+(2, \mathbb{R})$ on $\Omega\mathfrak{M}_g$ can provide us with precious information on dynamical behavior of individual translation surfaces, as well as the geometry and topology of the moduli space $\mathfrak{M}_g$, it also has profound connections with numerous domains such as the dynamics of billiards in rational polygons, Teichmüller theory (geometry, topology, and dynamics in Teichmüller space), interval exchange transformations...

The questions that I am currently interested in concern the classification of $\mathrm{GL}^+(2, \mathbb{R})$-orbit closures in various strata, geometric and dynamical properties of translation surfaces called *Prym eigenforms* which are discovered by McMullen, topology of strata and $\mathrm{GL}^+(2, \mathbb{R})$-orbit closures. I am also interested in related domains such as Mapping Class Groups (for instance, which subgroups of the MCG can be realized as the stabilizer of a Techmüller disc), spaces of different geometric structures on surfaces, representations of surface groups into Lie groups, moduli space of flat metric structures on surfaces with non-trivial linear holonomy...

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Henning Niesdroy Bielefeld University

Geometric reduction theory

One year ago I started my Ph.D. studies under supervision of Prof. Dr. Kai-Uwe Bux. Before I graduated, I was particularly interested in topology and group theory. In order to combine these two topics I joined the group of my supervisor to work in geometric group theory.

In the following, I give a short introduction of what my thesis is about:

Consider $\mathrm{SL}_n(\mathbb{R})$, the hyperbolic space $\mathbb{H}^n := \mathrm{SL}_n(\mathbb{R})/\mathrm{SO}_n(\mathbb{R})$ and the action of $\mathrm{SL}_n(\mathbb{Z})$ on $\mathbb{H}^n$. Reduction theory describes a fundamental domain $\mathrm{S}_n \subset \mathbb{H}^n$.

Generalizing reduction theory to s-arithmetic groups, Godement found an adelic formulation treating all places simultaneously. Let K be a global number field, $\mathcal{G}$ be a reductive group (think of SL_n), and let $\mathbb{A}$ be the ring of adeles of K. Then $\mathcal{G}(K)$ is discrete in $\mathcal{G}(\mathbb{A})$, and Godement finds a fundamental domain (coarsly) for the action of $\mathcal{G}(K)$ on $\mathcal{G}(\mathbb{A})$. Later, Behr and Harder transferred this to the case when K is not a global number field, but a function field.

In 2012 Bux-Köhl-Witzel gave a geometric reformulation of Behr-Harder. Now the natural question arises, whether this geometric reformulation can be transferred back to the case of a number field. My task is to actually do that.

Further we just started a new project in our group. The question is, whether $\mathrm{SL}_2(\mathbb{Z}[t, t^{-1}])$ is finitely generated or not. This question has been open for more than thirty years, though I cannot tell who came up with it first.

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Damian Osajda Universität Wien

Combinatorial nonpositive curvature

In a joint project with J. Chalopin, V. Chepoi, and H. Hirai we explore various notions of *combinatorial nonpositive curvature.* This refers to local combinatorial conditions on cell complexes that imply nonpositive-curvature-like properties of their universal covers. One of the goals is to understand groups acting nicely on such complexes. Examples of combinatorially nonpositively curved complexes are: *CAT(0) cubical complexes* [2], i.e. simply connected cubical complexes with flag links; and *systolic complexes* [3], i.e. simply connected simplicial complexes with 6–large links.

A common generalization of the two above classes — *bucolic complexes* — was investigated in [1]. One-skeleta of bucolic complexes are the so-called *weakly modular graphs.* Currently, we are studying various nonpositive-curvature-like aspects of such graphs. It includes: a local-to-global characterization, the quadratic isoperimetric inequality, conditions implying hyperbolicity. For certain subclasses of graphs and associated complexes we obtain stronger results, e.g. existence of a canonical CAT(0) metric (not true in general, even for systolic complexes).

The main advantage of such combinatorial approach is a possibility of constructing new examples of groups with interesting properties, e.g. new high dimensional hyperbolic groups, cf. [3,4,5]. On the other hand one can use the theory to obtain new results for classical groups, cf. [5].

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Wenyu Pan Yale University

Counting and equidistribution problems for orbits of thin groups in homogeneous spaces

My current research is the study of counting and equidistribution problems for orbits of thin groups in homogeneous spaces.

Let G denote the identity component of the special orthogonal group $\mathrm{SO}(n,1)$, $n \geq 2$. Let V be a finite dimensional real vector space on which G acts linearly from the right. Fix $v_0 \in V$ and a subgroup H of G , let $H_{v_0} = \{h \in H : \ v_0 h = v_0\}$ denote the stabilizer of v_0 in H. Suppose Γ is a discrete subgroup of G such that the orbit $v_0\Gamma$ is discrete. We want to understand the asymptotic behaviour of $\#\{v \in v_0\Gamma : \ \|v\| \leq T\}$, where $\|\cdot\|$ is a norm on V.

If Γ is a lattice, this problem is well understood. For example, when G_{v_0} is symmetric and Γ_{v_0} is a lattice in G_{v_0}, the value of $\#\{v \in v_0\Gamma : \ \|v\| \leq T\}$ is asymptotically proportional to the volume of $\#\{v \in v_0G : \ \|v\| \leq T\}$ (Duke et al 1993 [1]).

In Oh-Shah [2], they extend the problem to a suitable class of discrete subgroups Γ of infinite volume in G ; namely , the groups Γ with finite Bowen-Margulis-Sullivan measure m^{BMS} on $\Gamma\backslash\mathbb{H}^n$, where $\mathbb{H}^n$ is the n-dimensional hyperbolic space.

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Model theory of the free group

Model theory is the study of first-order formulas on a given language, which depends on the kind of structure studied (e.g.: groups, rings, ordered sets, ...). The simplest kind of first-order formula in the language of groups are equations (for example "$xy = yx$"), but in general one can also consider inequations (e.g. "$x^3 \neq 1$"), use logic connectors "OR" and "AND" (e.g. "$x^3 = 1$ AND $x \neq 1$") as well as quantifiers on the elements of the group (e.g. "$\forall x\ \forall y\ xy = yx$"). In general, some of the properties of a group or of its elements can be expressed by first-order formulas (such as abelianity, k-nilpotency, etc.), but others cannot (e.g. being finitely generated): the set of all first-order formulas satisfied by a group is called its first-order theory. The long standing Tarski problem asked whether free groups of different ranks have distinct first-order theory, in other words whether first-order formulas can detect the rank of a free group. The question was finally answered in the negative by Sela and independently by Kharlampovich and Miasnikov, who showed that (apart from the free group of rank one, which is abelian), all the free groups have the same first order theory.

The tools developped by Sela to tackle this problem relied heavily on geometric groups theory concepts such as the JSJ decomposition for hyperbolic groups, Rips theory for actions on real trees, etc. The results themselves show strong geometric connections: for example, the fundamental group of a closed surface of characteristic at most -2 has the same first-order theory as the free groups. This connection proved very useful in answering other model-theoretic questions about free and hyperbolic groups, and this has been my main research topic so far.

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Catherine Pfaff Université d'Aix-Marseille

$Out(F_r)$ Invariants and the Geometry and Dynamics of Culler-Votmann Outer Space

I study outer automorphisms of free groups. For a rank-r free group F_r, the *outer automorphism group* $Out(F_r)$ consists of equivalence classes of automorphisms of F_r, with automorphisms differing by an inner automorphism deemed equivalent. I have thus far focused on the most common elements of $Out(F_r)$, namely the *fully irreducibles*. I have developed machinery to construct fully irreducibles with certain invariants (namely index lists and ideal Whitehead graphs), as well as for proving when such examples cannot exist.

As with SL(2,R) acting on hyperbolic space, a central method for studying mapping class groups has been to study the action of each mapping class group on its Teichmüller space. To extend this analogy to the $Out(F_r)$, Culler and Vogtmann [CV86] constructed a topological space CV_r, *outer space*, on which $Out(F_r)$ acts properly with finite stabilizers. Some behavior of this action is much like in hyperbolic or mapping class group settings. For example, Levitt and Lustig [LL03] proved each fully irreducible $\phi \in Out(F_r)$ acts with North-South dynamics on the natural compactification $\overline{CV_r}$ of CV_r. On the other hand, Handel and Mosher [HM11] proved that, instead of a fully irreducible acting on CV_r with a unique axis, as does a loxodromic isometry acting on hyperbolic space, in many cases the axis for a fully irreducible is not unique. Handel and Mosher define [HM11], for a nongeometric fully irreducible $\phi \in Out(F_r)$, the axis bundle $\mathcal{A}_\phi$, an analogue of the Teichmüller axis for a pseudo-Anosov or the axis for a loxodromic isometry acting on hyperbolic space. Together with Lee Mosher, in [MP13], we prove precisely when a fully irreducible behaves more like a pseudo-Anosov or loxodromic isometry in that its axis bundle is a single axis. The result, in fact, illuminates a setting where one can actually quite easily identify when two fully irreducibles are conjugate in $Out(F_r)$.

In addition to expanding my invariant realization results, much of my current research focuses on using the automata I used to construct fully irreducibles to instead construct geodesics and points on the boundary of Outer Space with various properties.

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Representations of copolarity one

In representation theory one important invariant is the orbit space. Let W be a finite dimensional vector space and H a connected, compact Lie group with an effective representation $\rho : H \to O(W)$. In this way we can regard H as a group acting on the space W. The orbits of H decompose the space W in equidistant submanifolds. The orbit space H/W inherits a natural metric induced by the distance of the orbits. Now assume there is another representation $\tilde{\rho} : G \to V$ of a compact group G of lower dimension $\dim G < \dim H$, such that the orbit spaces are isometric. We call such a representation (G, V) a reduction of (H, W).

Which properties of H can be transferred to G? Are there special properties if the reduction (G, V) is minimal with respect to the dimension of G?

The first question has some positive answers. For example if (H, W) has non trivial fixed points, this set forms a maximal Euclidean subspace in the quotient. Hence (G, V) is forced to have a set of fixed points of the same dimension. If the representation of H is irreducible, it can be shown that the same is true for the representation G. For a polar action the minimal reduction is given by a discrete group G. I am interested in representations admitting a 1-dimensional minimal reduction. Therefore, I study S^1-representations which admit special properties arising from the fact that they are assumed to be minimal reductions of some higher dimensional representation. In the case of (H, W) being an irreducible representation Lytchak and Gorodski [1] showed that the representation is of codimension 3, i.e. a maximal dimensional orbit has codimension 3. In the reducible case some similar results can be obtained. I proved that, in most cases, the representation splits in a codimension 3 representation and a polar one. To get some further general results on representations which admit minimal reductions of low dimension, say $\dim G = \{2, 3\}$, I am interested in low dimensional and discrete groups.

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Homotopy theory of Davis-Januszkiewicz spaces

Given a finite simplical complex K and a pair of spaces (X, A), one constructs a topological space called the (generalized) moment-angle complex $Z(K,(X,A))$. The Davis-Januszkiewicz space of a complex K can be defined as $\mathcal{DJ}(K) = Z(K, (\mathbb{C}P^\infty, *))$. It has been shown that there are many connections between combinatorial properties of K and homotopy-theoretical properties of $\mathcal{DJ}(K)$, e.g. existence of vertex colorings of K and splitting of certain line bundles over $\mathcal{DJ}(K)$. I have just started my PhD, and I will be studying these spaces under supervision of Jesper M. Moller.

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Cocompactly cubulated graph manifolds

joint with Mark F. Hagen

A *graph manifold* is a compact oriented aspherical 3–manifold M that has only Seifert-fibred blocks in its JSJ decomposition. We say that a torsion-free group is (*cocompactly*) *cubulated* if it is the fundamental group of a (compact) nonpositively curved cube complex. A (cocompactly) cubulated group is (*compact*) *special* if the complex is (compact) *special*, i.e. admits a local isometry into the Salvetti complex of a right-angled Artin group.

Liu proved in [3] that if a graph manifold M admits a nonpositively curved Riemannian metric, then $\pi_1 M$ is virtually cubulated (and in fact special). Under the stronger hypothesis that M has nonempty boundary, the same conclusion was obtained in [4]. However, the resulting cube complex was in general not compact.

In our work we answer Question 9.4 of Aschenbrenner, Friedl, and Wilton [1] by characterizing graph manifolds M with $\pi_1 M$ virtually cocompactly cubulated, i.e. having a finite-index subgroup that acts freely and cocompactly on a CAT(0) cube complex. We show, moreover, that whenever this is the case, $\pi_1 M$ is virtually compact special.

We note that if M has no JSJ tori, i.e. M is Seifert-fibred, then by [2, Thm 6.12] the group $\pi_1 M$ is cocompactly cubulated if and only if the Euler number of the Seifert fibration vanishes. In this situation, the cube complex can easily be seen to be virtually special using [5]. If M is a Sol manifold, then $\pi_1 M$ is not cocompactly cubulated

We therefore assume that M is not a Sol manifold and has at least one JSJ torus, so that its underlying graph $\Gamma = (V, E)$ has at least one edge. We also assume that M does not contain π_1–injective Klein bottles, so that the base orbifolds of all Seifert-fibred blocks are oriented and hyperbolic. For each $v \in V$, we denote by $B_v \subset M$ the corresponding Seifert-fibred block, and for each edge $e \in E$, we denote by T_e the corresponding JSJ torus. For an edge e incident to v, let $Z_v^e \subset T_e$ be an embedded circle that is a fiber in B_v.

Definition. *A graph manifold M is* chargeless *if for every block B_v we can assign integers n_e to all edges $e = (v, v')$, so that in integral homology $\mathsf{H}_1(B_v)$ we have $\sum_e n_e [Z_{v'}^e] = 0$.*

In other words, a graph manifold is chargeless if in each block there is a horizontal surface whose boundary circles are vertical in adjacent blocks. Note that if M' is a finite cover of a graph manifold M, then M' is chargeless if and only if M is chargeless. Our first result is the following.

Theorem. *Let M be a chargeless graph manifold. Then $\pi_1 M$ is virtually compact special.*

This theorem is one of few results giving an obstruction to being cocompactly cubulated for a specific class of groups. Another notable result of this type is Wise's characterization of tubular groups that are cocompactly cubulated [6, Thm 5.8]. Our main theorem is the following converse.

Theorem. *Let M be a graph manifold. If $\pi_1 M$ is virtually cocompactly cubulated, then M is chargeless.*

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Free groups: word maps, representations and growth

Many of the lines of research I have followed and am currently following are related to the study of free groups (other topics include mainly expansion of random graphs and Ramanujan graphs). Let me mention the main questions related to free groups which I have considered so far:

Measure-theoretic characterization of words:
Consider the measure induced on finite groups by words. Namely, fix some word $w \in F_k$, the free group on k generators $x_1, \ldots, x_k$. This word induces a measure on every finite group via the word map $w : G^k \to G$ (here G^k is the Cartesian product of G) and a push forward of the uniform measure on G^k. (Put differently, for each $1 \leq i \leq k$, substitute x_i with an independent, uniformly distributed random element of G and evaluate the product defined by w to obtain a random element in G.) It is an easy observation that primitive words, namely words belonging to some basis of F_k, induce the uniform measure on every finite group G. Several mathematicians have conjectured that this property actually characterizes primitive elements, i.e. that a word which induces the uniform measure on every finite group is primitive. In [1] (to appear in Israel Journal of Mathematics) I have proven the conjecture for F2, and in [2] (to appear in the Journal of the AMS), together with a fellow student, Ori Parzanchevski, we proved the conjecture in full.

The most interesting open question in this line of research is the following. The (now resolved) question about primitive words is actually a special case of a more general problem. The primitives constitute a single $Aut(F_k)$-orbit in F_k. In the same manner that they induce the same measure on every finite (or compact) group, it is an easy observation that any two words belonging to the same $Aut(F_k)$-orbit induce the same measure on every finite (compact) group. But does the converse hold? Namely, if w_1 and w_2 belong to different orbits, is there necessarily some (finite? compact?) group on which they induce different measures?

The role of primitives in Representations of Free groups:
There is a family of challenging questions and conjectures revolving around (finite-dimensional) representations of free groups and their automorphism groups. I am currently working on some of these questions. Procesi and Formanek have shown that $Aut(F_k)$ is non-linear for $k \geq 3$ [FP92]. Their proof suggests the following stronger statement: For $k \geq 3$, in any representation

$\rho : Aut(F_k) \to GL_d((C))$, the image of $Inn(F_k) \cong F_k$ is virtually solvable [Lub11, Conj. 6.12]. It can be shown (Gelander) that this conjecture follows from the following one: If $\rho : F_k \to G$ is a representation into a complex simple algebraic group with $\rho(F_k)$ dense in G, then already $\rho(P_k)$ is dense, where P_k is the set of primitive elements in F_k. This explains some of the motivation to the following concrete question which we are studying:

Let $\rho : F_k \to GL_d((C))$ be a finite-dimensional representation of the free group F_k. To what extent do primitive words determine the (character of the) representation? More concretely, can there be two different characters of F_k which coincide on the set of primitives P_k?

Growth and Asymptotics in Free Groups:
I have recently studied a question about the number of primitive elements in F_k. Let $P_{k,N}$ be the number of primitive elements of length N in F_k. It has been well known, at least for a decade now, that primitives are rare, in the sense that a (uniformly distributed) random word of length N is a.a.s. non-primitive as $N \to \infty$. Moreover, the decay is exponential: several upper bounds were found for $\limsup_{N\to\infty} \sqrt[N]{P_{k,N}}$, the exponential growth rate of primitives(e.g. [BMS02b]). (Recall that the total number of words of length N is roughly $(2k-1)^N$). Yet the exact growth rate of primitives remained unknown. Attributed to M. Wicks, this question was the content of one of the open problems in [BMS02a]. In [PW14], together with Conan Wu, a PhD student from Princeton University, we answer the question and show that the exact exponential growth rate of primitives is $2k-3$. Moreover, we show a somewhat surprising result about a generic primitive element: It turns out that most primitives are words which are “obviously” primitive, namely, words which, up to conjugation, contain one of the letters exactly once. The proof is based on a meticulous analysis of Whitehead's algorithm to detect primitive words.

This work has several follow-up questions. For example, what is the growth rate of other $Aut(F_k)$-orbits in F_k? Which orbits, primitives aside, have the largest growth? What is the growth of $Aut(F_k)$ w.r.t. standard generating sets such as Nielsen moves or Whitehead automorphisms?

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Mapping tori over right-angled Artin groups

The algebraic mapping torus of the automorphism $\phi : G \to G$ is the group $M_\phi = \langle G, t : t^{-1}xt = \phi(x), \forall x \in G\rangle$, or $G \rtimes_\phi \mathbb{Z}$. These groups arise naturally in topology as fundamental groups of topological mapping tori, and in group theory as ascending HNN extensions. The simplest and most geometrically relevant cases are those in which the bases are (finitely generated) free abelian groups, surface groups, and free groups. Indeed these three types of bases are well studied. It is known that if G is free then M_ϕ is coherent, [1] that is finitely generated subgroups are finitely presentable. By direct analysis, Bridson and Groves proved that M_ϕ is either hyperbolic or has quadratic Dehn function when G is free [2].

Right-angled Artin groups (RAAGs), also called partially commutative groups, are groups on n generators in which the relations are commutators of generators. The free groups and free abelian groups are the two extremes of this spectrum of bases, but such interesting groups as $F_2 \times F_2$ are also examples of RAAGs. These groups have lots of nice structure which makes them good to work with. For example, they have finite complete rewriting systems, a tool for easily rewriting words into normal forms [3]. RAAGs have appeared in many applications, as a source of examples and counterexamples, and their actions on CAT(0)-cube complexes make them a popular tool in geometric group theory. For instance, RAAGs were an important tool in Agol's solution to the Virtual Haken Conjecture.

I am interested in mapping tori which have right-angled Artin base group. In particular I wish to understand the Dehn functions of these groups and their finiteness properties. At the moment these problems appear to be out of reach for general base RAAGs, so I am working on special cases.

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Boundaries of CAT(0) Space

My research focuses on the boundaries of CAT(0) spaces, specifically, finite dimensional, locally finite, cocompact CAT(0) cube complexes with rank-1 isometries. I give an explicit and elementary proof that the Croke-Kleiner space [1] does not have unique G-equivariant boundary if we fix the gluing angle at $\pi/2$ and changes the translate lengths of the generators of its fundamental group. I also prove that right-angled Coxeter groups do not have unique G-equivariant boundary. Currently I am studying the connection between Roller boundary [2] and visual boundary for rank-1, finite dimensional, locally finite, cocompact CAT(0) cube complexes. The study aims to find a "nice" map between the two that will extend the known properties of the Roller boundary. I am also highly interested in coarse median spaces [3] and would like to make connections between coarse median metric and CAT(0) metric.

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Limit multiplicities problems and geometry

Broadly speaking, I am interested in the relations between global geometric invariants (such as the volume) and topological invariants for locally symmetric spaces, especially hyperbolic manifolds. Particularly interesting examples among those are the so-called arithmetic ones, and I will now try to explain a specific problem in the vein mentioned above for this class, the so-called limit muliplicities problem (which originally comes from the theory of automorphic forms, but is also of interest in relation to spectral geometry and topology).

Let G be a real simple noncompact Lie group. The classical limit multiplicities (LM) problem asks to show that, for a sequence Γ_n of nested congruence subgroups of a given arithmetic lattice Γ, the multiplicities of irreducible representations of G in $L^2(\Gamma_n \backslash G)$ (a sequence of atomic measures on the unitary dual) approximate, when renormalized by the volume of $\Gamma_n \backslash G$, the Plancherel measure of G. This may be a little unintelligible to a geometrically-oriented reader, but a nice consequence of it is the following limit for the Betti numbers of the Γ_n:

$$\lim_n \frac{b_p(\Gamma_n)}{\mathrm{vol}(\Gamma_n \backslash G)} = \begin{cases} \frac{\chi(\Gamma)}{\mathrm{vol}(\Gamma \backslash G)} & \text{if } p = \dim X/2 \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Let the reader know that a general solution to LM is not yet known in the nonuniform case (for uniform lattices it has been solved in the work of Delorme for principal congruence subgroups and Abert–Bergeron–Biringer–Gelander–Nikolov–Samet and myself [1] in general), but there has been recent progress in the case where $G = \mathrm{SL}_n(\mathbb{R})^m$ by Finis–Lapid–Müller [2]. I am personally more interested in the following questions, which take the problem in a different direction:

(a) Does (1) hold for sequences of noncommensurable lattices ? (Note that given G, a result of I. Samet asserts that $b_p(\Gamma)/\mathrm{vol}(\Gamma \backslash G)$ is bounded independently of Γ.)

(b) There should be results similar to (1) for the torsion part of the homology groups — for example, for a sequence Γ_n of congruence subgroups of an arithmetic lattice in $G = \mathrm{SL}_2(\mathbb{C})$, we expect that

$$\lim_n \frac{\log |H_1(\Gamma_n)_{\mathrm{tors}}|}{\mathrm{vol}(\Gamma_n \backslash G)} = \frac{1}{6\pi} \tag{2}$$

holds.

For more on (b) the reader should look at the work of Bergeron–Venkatesh [3], and my own paper [4] for the nonuniform case. Regarding (a) in rank one (the higher-rank case is dealt with in [1]) an interesting special case is that of sequences of maximal arithmetic lattices. Indeed, it seems that the methods used in [1] to deal with arbitrary sequences of congruence subgroups could generalize to some such sequences; for example in [5] it is proven that if Γ_n is a sequence of hyperbolic three-manifolds obtained from maximal orders in quaternion algebras defined over imaginary quadratic fields (for example $\Gamma_n = \mathrm{SL}_2(\mathbb{Z}[\sqrt{-d_n}])$ for a sequence of square-free d_n) then (1) holds. The proof of this result — as those in [1] — rests on establishing the convergence of the sequence to the universal cover $\mathbb{H}^3$ in a certain geometric sense. This is a result of interest in its own right and it would be interesting to generalize it to higher-dimensional real hyperbolic manifolds or complex hyperbolic ones.

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Statement of research

I'm a PhD student under the joint supervision of Étienne Ghys and Yulij Ilyashenko. The broad name for my research is 'The dynamics of physical systems'. The research finds motivation in physical systems with interesting behaviour and uses mathematical tools to explore them: primarily the methods of dynamical systems coupled with those of geometry. There are three main directions in which I am currently working: the dynamics of Josephson equation, search of open classes of diffeomorphisms of manifolds with boundary possessing thick attractors and the study of integrable mechanical systems, especially those with close orbits.

First, Josephson equation is a vector field on a 2–torus modelling a system with a Josephson junction. This vector field arises from a family of differential equations on a circle $\mathbb{R}/2\pi\mathbb{Z}$ of the following form

$$\frac{dx}{dt} = \frac{\cos x + a + b\cos t}{\mu} \tag{3}$$

The aim of my work is to describe some limit properties of Arnold tongues for (3) in dependence of the parameters. For $\mu \ll 1$ the methods of the fast–slow systems theory can be applied in order to study the behaviour of such a system, see [1], [2]. For $b \gg 1$ the asymptotic behaviour of Arnold tongues boundaries in terms of Bessel functions can be discovered, see [3].

Second, I am working on generalizing the results of [4] for a wider class of 'thick' attractors on the manifolds with boundary. I'm trying to elaborate the techniques analogous to a classical Sternberg theorem of linearization in the case of skew products preserving the special structure of a diffeomorphism. The obtained normalization theorems are supposed to be applied to the construction of a wide class of so called 'thick' attractors of dynamical systems on the manifolds with boundary.

Third, I am searching for some new cases of integrable systems as well as for systems with potentials giving closed orbits. The inspiration is a classical Bertrand's theorem that states that in the case when the force only depends on the distance to the center of attraction, all the bounded velocity trajectories are periodic only for the newtonian and harmonic potentials. In a similar way, Zoll surfaces present wonderful examples of manifolds on which the geodesic flow is periodic. One can also mention the marvelous Kowaleski top exhibiting a completely integrable dynamics and more recent results, as for example a spherical ball rolling inside a vertical cylinder and submitted to its weight [5]. In this case, the ratio of two frequencies of the motion turns out to be independent of initial conditions. The purpose of my research is to analyze these examples and to understand them in a common framework. On the way, I hope to discover new interesting examples.

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Cube complexes and virtual fibering of 3-manifolds

I am a first year PhD student in Bologna, under the supervision of Stefano Francaviglia and I am mainly interested in Geometric Group Theory for its applications to low-dimensional geometry and topology. The work of Thurston and Perelman has shown that the study of 3-dimensional manifolds can be reduced to the hyperbolic case, where Mostow's Rigidity Theorem is available. This theorem essentially says that the geometry of a hyperbolic manifold in dimension 3 (or higher) is completely determined by its fundamental group, so it makes perfectly sense to switch attention from the properties of the manifold to those of the group.

Through the recent work of Agol and Wise, some of the geometric properties of these groups (e.g. hyperbolicity) have proven to be crucial in the solution of two longstanding problems in low dimensional topology (Virtually Haken and Virtually Fibering Conjectures) about the existence and structure of suitable surfaces in (a finite cover of) a hyperbolic 3-manifold.

One of the main ingredients in the proofs of the above conjectures has been the construction of a non-positively curved cube complex with the same fundamental group as the hyperbolic manifold and some additional properties about the intersection patterns of its hyperplanes (= codimension-1 subspaces), known as speciality conditions. These allow to get a virtual embedding of the fundamental group into a right-angled Artin group, from which it inherits some useful algebraic properties.

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Geometry of the conjugacy problem

Let G be a finitely presented group with finite generating set A. The *word problem* on G asks whether there is an algorithm to determine when a word on $A \cup A^{-1}$ represents the identity element of G. The *Dehn function* of G represents the geometric complexity of the word problem. It measures the minimal area required to fill a loop in the Cayley 2–complex of G and to estimate it is an *effective* version of the word problem. Determining the Dehn function of groups has been a fundamental question in geometric group theory.

The word problem is a special case of the *conjugacy problem*, which is solvable in G if we can write an algorithm which, on input two elements of G, determines if they are conjugate. A naturally related question is the so-called *effective conjugacy problem*, which quantifies the conjugacy problem. The word length of an element g in G is the minimal length of a word on $A \cup A^{-1}$ representing g. The *conjugacy length function* gives a measure of the minimal length of an element g satisfying the relationship $ag = gb$ for a, b in G, relative to the sum of the lengths of a and b.

Free solvable groups and wreath products. In [2], I studied the behaviour of conjugacy length under wreath products and what this means for free solvable groups. For a wreath product $A \wr B$, I related the length of short conjugators in the wreath product to the conjugacy length function in B and the distortion of the word length in cyclic subgroups in B. The conjugacy length function of A can also appear, but only if the group B contains torsion elements. In particular, for free solvable groups, denoted $S_{r,d}$, where r is the number of free generators and d is its derived length, I obtained the following result:

Theorem 1 ([2]). *Let $r, d > 1$. Then the conjugacy length function of the free solvable group $S_{r,d}$ is bounded above by a cubic polynomial.*

An important step in the proof of this Theorem concerns the Magnus embedding. If N is a normal subgroup of a (non-abelian) free group F of rank r, whose derived subgroup is denoted N', then the Magnus embedding expresses F/N' as a subgroup of the wreath product $\mathbf{Z}^r \wr F/N$. In particular, if we take N to be the d–th derived subgroup of F then we obtain an embedding of $S_{r,d+1}$ into $\mathbf{Z}^r \wr S_{r,d}$.

Theorem 2 ([3]). *The Magnus embedding $\varphi : F/N' \hookrightarrow Z^r \wr F/N$ is a bi-Lipschitz embedding.*

Lattices in semisimple Lie groups. While the conjugacy problem applies to recursively presented groups, the effective conjugacy problem requires only that the group admits a metric. A long-term project I am working on is to understand the conjugacy length function of lattices in a higher rank semisimple real Lie group G. The following two results are for the ambient Lie group, but they have consequences for conjugacy in the lattice.

A real hyperbolic element of G is an element which, in the associated symmetric space G/K, translates a biinfinite geodesic and all geodesics parallel to it. Given such an element $a \in G$, the slope of a corresponds, roughly speaking, to the location in a Weyl chamber of a geodesic translated by a. A more accurate definition can be found in [1].

Theorem 3 ([1]). *Let G be a higher-rank real semisimple Lie group. Let a, b be conjugate real hyperbolic elements in G of slope ξ. Then there exists a conjugator $g \in G$ such that*

$$d_G(1, g) \leq \ell_\xi(d_G(1, a) + d_G(1, b))$$

where ℓ_ξ is a positive constant depending on the slope ξ.

The second type of element I looked at are *unipotent elements*. These are elements which in some finite-dimensional, faithful, linear representation are conjugate to an upper triangular matrix with 1's on the diagonal. For these elements we apply a more algebraic approach. We look at pairs of conjugate unipotent elements for whom, in the matrix representing them, the superdiagonal entries are bounded away from zero by some $\delta > 0$. We describe these elements as having *simple entries* of size at least δ. Provided that the Lie algebra of G is split we can obtain the following:

Theorem 4 ([1]). *Fix $\delta > 0$. There exists $L > 0$ depending on δ such that, if u and v are conjugate unipotent elements in G whose simple entries all have size at least δ, then we can construct $g \in G$ such that $gug^{-1} = v$ and which satisfies*

$$d_G(1, g) \leq L(d_G(1, u) + d_G(1, v)).$$

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Flat Lorentzian structures on 3-manifolds

My research so far has been focused on the study of maximal globally hyperbolic flat Lorentzian 3-manifolds, supervised by Francesco Bonsante. A MGH flat Lorentzian manifold M is a 3-manifold endowed with a flat pseudo-Riemannian metric (of signature 2,1) with some good properties related to causality. For example, M is homeomorphic to $S \times \mathbb{R}$, where S is a closed surface of genus $g \geq 2$. Classification results of such space-times, with a prescribed topology, were provided by Mess. In particular, their moduli space was described:

Theorem (Mess)**.** *If S is a surface of genus $g \geq 2$, future-complete MGH flat Lorentzian structures on $S \times \mathbb{R}$, up to isometry isotopic to the identity, are parametrized by the tangent bundle $T\,Teich(S)$ of the Teichmüller space of S.*

We have extended such result to spacetimes and surfaces with cone singularities. A hyperbolic surface is said to have a cone singularity of angle $\theta < 2\pi$ at a point if the local model at that point is a wedge in $\mathbb{H}^2$, where the edges of the wedge are glued by a rotation. An analogous definition holds in Minkowski space, the wedge being the intersection of two half-planes, and the rotation being around a time-like line. The techniques used by Mess do not apply to this case, but by using geometic-differential techniques we have reobtained Mess' result and extended it to the singular case.

Theorem. *If S is a surface of genus $g \geq 2$ with n cone singularities of angles $\theta_1, \ldots, \theta_n < \pi$, future-complete MGH flat Lorentzian structures with n cone singularities along time-like lines of angles $\theta_1, \ldots, \theta_n$ on $S \times \mathbb{R}$, up to isometry isotopic to the identity, are parametrized by the tangent bundle $T\,Teich(S)_{\theta_1,\ldots,\theta_n}$ of Teichmüller space of hyperbolic surfaces with cone singularities of fixed angles.*

During my PhD, I plan to continue research on such spacetimes and, more in general, study several types of Lorentzian structures on 3-manifolds.

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Tilings of affine space

I study discrete subgroups of the affine group $\mathrm{GL}_n(\mathbb{R}) \ltimes \mathbb{R}^n$ that act properly discontinuously on the affine space $\mathbb{R}^n$. In the case where they preserve a Euclidean norm, their behaviour is well-known. In particular, we have the following classical result:

Theorem (Bieberbach 1911):

- Every $\Gamma \subset \mathrm{O}_n(\mathbb{R}) \ltimes \mathbb{R}^n$ with properly discontinuous action is virtually abelian.
- Every $\Gamma \subset \mathrm{O}_n(\mathbb{R}) \ltimes \mathbb{R}^n$ with properly discontinuous and cocompact action is virtually isomorphic to $\mathbb{Z}^n$

(we say that Γ *virtually* has some property if it has a finite index subgroup with this property).

In the general case, two conjectures were made trying to generalize this result:

Conjecture (Auslander 1964): Every $\Gamma \subset \mathrm{GL}_n(\mathbb{R}) \ltimes \mathbb{R}^n$ with properly discontinuous and cocompact action is virtually solvable.

Conjecture (Milnor 1977): Every $\Gamma \subset \mathrm{GL}_n(\mathbb{R}) \ltimes \mathbb{R}^n$ with properly discontinuous action is virtually solvable.

To put these statements into proper context, we remind the following theorem:

Tits' alternative: Every finitely generated linear group is either virtually solvable or contains a free subgroup of order 2.

In 1983, Margulis disproved Milnor's conjecture by constructing a free group of affine transformations acting properly discontinuously, with linear part Zariski-dense in $\mathrm{SO}(2,1)$ (see [4]). My research focuses on such counterexamples: I try to construct as many of them as possible and to study their properties. If I am ever able to completely classify them, this would lead to an answer to the Auslander conjecture. For a survey of already known results, see [1].

My first result dealt with a generalisation of the Margulis counterexample. In 2002, Abels, Margulis and Soifer found free, properly discontinuous affine groups with linear part Zariski-dense in $\mathrm{SO}(d+1,d)$, for any odd d (see [2]). In [5], by adapting an approach developed by Drumm for $d = 1$ (see [3]), I explicitly constructed a fundamental domain for these groups, which shed light on the topology of the quotient manifold.

Right now, I am trying to construct a new family of free properly discontinuous affine groups, with linear part Zariski-dense in some Lie groups G and acting on its Lie algebras $\mathfrak{g}$ by the adjoint action.

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Boundaries and surface group amalgams

I am interested in the visual boundary of hyperbolic groups, quasi-isometry and commensurability classes, and the abstract commensurator of a group. My research focuses on hyperbolic surface group amalgams over $\mathbb{Z}$ and the class of limit groups.

A quasi-isometry between δ-hyperbolic spaces induces a homeomorphism between the visual boundary of these spaces, though the converse is false. For example, $\partial\mathbb{H}^4_{\mathbb{R}} \cong \partial\mathbb{H}^2_{\mathbb{C}} \cong S^3$, but $\mathbb{H}^4_{\mathbb{R}}$ and $\mathbb{H}^2_{\mathbb{C}}$ are not quasi-isometric. In general, to obtain a quasi-isometry via the boundary, one needs to exhibit a homeomorphism between visual boundaries that is a quasiconformal or quasi-Möbius equivalence. I am broadly interested in understanding for which classes of groups a homeomorphism between their visual boundaries is sufficient to prove the groups are quasi-isometric.

With this question in mind, I am studying the geometry of limit groups and groups in $\mathcal{C}$, the class of groups isomorphic to the fundamental group of two closed hyperbolic surfaces identified along a closed curve in each, and $\mathcal{C}_S$, the subclass in which the curves that are glued are simple. Groups in $\mathcal{C}$ are δ-hyperbolic, so the visual boundary may be used to study the quasi-isometry classes. For example, the boundary of a lift of the curves identified is a pair of global cut points in the boundary of the group, and the number of components in the complement of the boundary of a lift depends on whether the curves identified have simple representatives on the surface. One may use this to show not all groups in $\mathcal{C}$ are quasi-isometric. As part of my thesis, we show all groups in $\mathcal{C}_S$ are quasi-isometric by exhibiting a a bi-Lipschitz map between the universal covers of a $K(G,1)$ space for these groups, equipped with a CAT(-1) metric. I am studying the quasi-isometry classification within the broader class $\mathcal{C}$ and the class of limit groups, which intersects $\mathcal{C}$ [1].

The commensurability classes in $\mathcal{C}$ are finer than the quasi-isometry classes. Within $\mathcal{C}_S$, we characterize the commensurability classes in terms of the ratio of Euler characteristic of the two surfaces and the topological type of the curves identified. An important tool in the classification is the *topological rigidity* of these groups, which is addressed via the visual boundary in [2] and [3]. That is, any isomorphism between finite index subgroups of groups in $\mathcal{C}_S$ is induced by a homeomorphism between the $K(G,1)$ space consisting of closed surfaces identified along a set of closed curves. I would like to understand how this rigidity holds for limit groups and within $\mathcal{C}$, and I hope to understand the commensurability classes and abstract commensurator of groups in these classes.

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Algebraic properties of random groups

The outstanding Kaplansky zero-divisor conjecture states that the group ring of a torsion-free group over an integral domain has no non-trivial zero-divisors. The conjecture is known for

- Groups satisfying the unique product property,
- Torsion-free elementary amenable groups,
- Virtually compact special groups.

Delzant showed that Gromov hyperbolic groups which act with large translation length on a hyperbolic space satisfy the unique product property. Rips-Segev have constructed a torsion-free group without the unique product property.

We have obtained the following *results.*

- First examples of Gromov hyperbolic such groups.

- A Rips construction without the unique product property. (Joint with Goulnara Arzhantseva.)

Our groups without the unique product property have various algebraic and algorithmic properties. The presentations of such groups are not generic among random finitely presented groups.

My *main interest* is in the following two open questions:

- Is the unique product property generic among random finitely presented groups?
- Do the Rips-Segev groups without the unique product property satisfy the Kaplansky zero-divisor conjecture?

The *methods* I use go from small cancellation theories over free products to the Gromov graphical model as well as the Arzhantseva-Ol'shanskii few relator model for random groups.

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Linear locally compact simple groups

The goal of this research project is to obtain some classifications within the class $\mathcal{S}$ of locally compact, compactly generated and topologically simple groups. The study of this class is motivated by the results in [1], which show that the groups in $\mathcal{S}$ act as elementary blocks in the structure theory of compactly generated groups.

The main known examples of groups in $\mathcal{S}$ are: simple Lie groups, simple algebraic groups over local fields, finitely generated simple groups, complete Kac-Moody groups over finite fields, some automorphisms groups of trees or of right-angled buildings, and some generalizations of the latter.

Among those examples, simple Lie groups, simple algebraic groups over local fields and finite simple groups admits a faithful continuous representation in some $GL_n(k)$, for k a locally compact field. Currently, we hope to prove that this property characterizes those groups in the class $\mathcal{S}$. To put it concisely, we hope to prove the following:

Conjecture *Let G be a group in the class $\mathcal{S}$. Assume that G has a continuous faithful linear representation in $GL_n(k)$, where k is a local field. Then*

G is a finite simple group, or a simple Lie group, or a simple algebraic group over a local field.

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Commutator Words in Mapping Class Groups

Mapping class groups have many flats, that is, subgroups homomorphic to $\mathbb{Z}^n$, hence are not δ-hyperbolic. But they act naturally on curve complexes, which are simplicial complexes with simple closed curves as vertices. Masur and Minsky [1] showed that the curve complex $\mathcal{C}(S)$ is δ-hyperbolic. In their second paper on curve complexes [2] they showed subsurface projections control the behavior of mapping classes. They also stated that subsurface projections have their own contraction property. They also solved the conjugacy word problem for pseudo-Anosov elements and Tao extended this result to every pair of conjugated elements in the mapping class groups [3].

Theorem ([2],[3]): *There exists $K > 0$. If g and f are conjugated pseudo-Anosovs, that is, $g = ufu^{-1}$, then there is $v \in MCG(S)$ such that $g = vfv^{-1}$ and that $|v| \leq K(|g| + |f|)$.*

This result leads to the following conjecture.

Conjecture *There exists $C, M > 0$, and $K > 1$ such that for any $w = [x_1, y_1] \cdots [x_k, y_k]$, if $|w| \geq M$, then there are $a_1, \ldots, a_k$, $b_1, \ldots, b_k$ which satisfy*

$$w = [a_1, b_1] \cdots [a_k, b_k]; \text{ and}$$

$$|w| \geq \frac{1}{K} \sum_{i=1}^{k} (|a_i| + |b_i|) - C.$$

Evidence that such a statement is true for δ-hyperbolic groups can be found in [4].

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Helly's theorem for systolic complexes

I am a first year PhD student at the Texas A&M University, interested in geometric group theory. I recently finished my master degree at the University of University of Warsaw, where my supervisior was Paweł Zawiślak. In my master thesis I proved the analogue of Helly's theorem for systolic complexes.

Recall classical Helly's theorem concerning convex subsets of Euclidean spaces. Suppose that $X_1, X_2, \dots, X_n$ is a collection of convex subsets of $\mathbb{R}^d$ (where $n > d$) such that the intersection of every $d+1$ of these sets in nonempty. Then the whole family has a nonempty intersection. This result gave rise to the concept of Helly dimension. For a geodesic metric space X we define its *Helly dimension* $h(X)$ to be the smallest natural number such that any finite family of $(h(X)+1)$-wise non-disjoint convex subsets of X has a non-empty intersection. Clearly, Helly's theorem states that Helly dimension of the Euclidean space $\mathbb{R}^d$ is $\leq d$. It is very easy to find examples showing that it is exactly equal to d.

Systolic complexes were introduced by Tadeusz Januszkiewicz and Jacek Świątkowski in [1]. They are connected, simply connected simplicial complexes satisfying some additional local combinatorial condition , which is a simplicial analogue of nonpositive curvature. Systolic complexes inherit lots of $CAT(0)$-like properties, however being systolic neither implies, nor is implied by nonpositive curvature of the complex equipped with the standard piecewise euclidean metric.

There is a well known result for $CAT(0)$ cube complexes which states that, regardless their topological dimension, they all have Helly dimension equal to one. This motivates a question about Helly-like properties of systolic complexes. We obtained the following results:

Let X be a 7-systolic complex and let X_1, X_2, X_3 be pairwise intersecting convex subcomplexes. Then there exists a simplex $\sigma \subseteq X$ such that $\sigma \cap X_i \neq \emptyset$ for $i = 1, 2, 3$. Moreover, the dimension of σ is at most two.

In other words 7-systolic complexes have Helly dimension less or equal to 1. It is easy to see that this is not necessarily true for 6-systolic complexes, but we prove that any systolic complex has Helly dimension less or equal to 2. More precisely:

Let X be a systolic complex and let X_1, X_2, X_3, X_4 be its convex subcomplexes such that every three of them have a nontrivial intersection. Then there exists a simplex $\sigma \subseteq X$ such that $\sigma \cap X_i \neq \emptyset$ for $i = 1, 2, 3, 4$. Moreover, the dimension of σ is at most three.

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Analysis in metric spaces

Most of my work is devoted to the study of the large scale geometry of different kinds of objects, among which: discrete groups, Lie groups, Riemannian manifolds, graphs, and more general metric measure spaces. It has connexion to topology (rigidity of topological manifolds), geometric group theory (e.g. word problem, via the study of the Dehn function), embeddings of (finite or infinite) metric spaces into Banach spaces... More specifically, here is a list of subjects I am working on.

A fair amount of my research has been devoted to the problem of quantifying, or characterizing how well metric spaces can be coarsely embedded into various classes of Banach spaces, most interestingly Hilbert spaces. For instance I gave a precise answer for free groups, and for Lie groups and their lattices. More recently, I have characterized those metric spaces which cannot be coarsely embedded in terms of containing a sequence of expanders (in some weak sense). My last work in the subject is a collaboration with Austin and Naor, where we prove sharp quantitative obstructions for the Heisenberg group to quasi-isometrically embed into a uniformly convex Banach space. Our estimates are new even in the case of Hilbert spaces.

A few years ago, I started a collaboration with Erik Guentner and Guoliang Yu on topological rigidity of manifolds. We developed new tools (involving some large-scale geometry), in order to prove the Stable Borel conjecture for a wide class of closed manifolds. We are now trying to mix our techniques with those of Farrel and Jones in order to attack the Borel conjecture.

In a recent work in collaboration with Yves Cornulier, we have been studying the 2-dimensional isoperimetric function on Lie groups. Our main result is an algebraic characterization of Lie groups with exponential/polynomial isoperimetric function. In particular we prove that such a dichotomy holds. We prove similar results for algebraic groups over a local field.

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Dale Winter Brown University

I'm interested in the ergodic theory of frame flow for geometrically finite rank one manifolds, and in applying that theory to the study of Lie groups. Here are two examples of the types of questions we try to answer:

Question 1: the orbit counting problem. Consider $G = \mathrm{PSL}_2(\mathbb{C})$ acting as the isometry group of $\mathbb{H}^3$, and let $\Gamma < G$ be a discrete subgroup without torsion elements. For a base point $z_0 \in \mathbb{H}^3$ we define the orbit counting function by

$$N(R, z_0) = \#\{\gamma \in \Gamma : d_{\mathbb{H}^3}(\gamma z_0, z_0) < R\}.$$

Clearly we should expect $N(R, z_0)$ to go to infinity as $R \to \infty$, but how fast? What are the asymptotics?

Question 2: measure classification for horospherical subgroups. Let Γ and G be as before, and let

$$N = \left\{ \begin{pmatrix} 1 & 0 \\ z & 1 \end{pmatrix} : z \in \mathbb{C} \right\} < G$$

be the subgroup of strictly lower triangular matrices. Classify all the locally finite Borel measures on $\Gamma \backslash G$ that are invariant and ergodic for the right N action.

These questions, and their natural generalizations, have good and influential answers when the quotient $\Gamma \backslash \mathbb{H}^3$ has finite volume; see [2] and [3] for the orbit counting problem, and [1] and [6] for the measure classification.

My own focus has been in trying to understand these questions for discrete subgroups $\Gamma < G$ that have a finite-sided fundamental domain (or more precisely, that are geometrically finite), but don't necessarily have finite covolume. There are a number of Radon measures on the frame bundle $\Gamma \backslash G$ that become important in this setting: the Bowen-Margulis-Sullivan measure m^{BMS}, which is finite and invariant for frame flow; the Burger Roblin measure m^{BR}, which is invariant for N; and the opposite BR measure m^{BR}_*, which is invariant for the opposite horospherical group N^*. The following theorem describes a special case of work of Roblin [7] and Flaminio-Spatzier [4], and gives some flavor of the results that can be proved.

Theorem (cf. [5], Theorem 2.5, Theorem 3.2, Theorem 4.4). *Suppose that $\Gamma < G = \mathrm{PSL}_2(\mathbb{C})$ is geometrically finite and Zariski dense (considering G as a real algebraic group). Let $\delta = \delta_\Gamma$ be the critical exponent of Γ. Then:*

- *the frame flow ϕ_t is mixing on $(\Gamma \backslash G, m^{\mathrm{BMS}})$;*
- *for any pair $f_i \in C_c(\Gamma \backslash G)$, the Haar measure matrix coefficients satisfy*

$$\lim_{t \to +\infty} e^{(2-\delta)t} \int_{\Gamma \backslash G} f_1(\phi_t(g)) f_2(g)\, dg = \frac{m^{BR}(f_1) m^{BR}_*(f_2)}{|m^{\mathrm{BMS}}|};$$

- *there is an explicit constant $c(z_0)$ such that*

$$\#\{\gamma \in \Gamma : d_{\mathbb{H}^3}(\gamma z_0, z_0) < R\} \sim c(z_0) e^{\delta t};$$

- m^{BR} *is the only* N*-invariant and -ergodic Radon measure on* $\Gamma\backslash G$ *that is not supported on a single closed* N *orbit.*

The key step here is to prove mixing of frame flow for $(\Gamma\backslash G, m^{\mathrm{BMS}})$. This, together with the product structure of the BMS measure, allows us to deduce the asymptotics for Haar measure matrix coefficients, and so to establish the orbit counting estimate. Mixing of $(\Gamma\backslash G, m^{\mathrm{BMS}})$ also implies the N-ergodicity of $(\Gamma\backslash G, m^{\mathrm{BR}})$, which is a key step in establishing the measure classification result.

My current project is to generalize these results from real hyperbolic spaces to other rank one symmetric spaces.

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Geometric structures on surfaces

My main area of research is the study of representation spaces of surface groups (or related groups) in $\mathrm{PSL}(2,\mathbb{R})$ (or related groups).

Holonomies of hyperbolic structures on a marked surface of genus $g \geq 2$ form a subset of $\mathrm{Hom}(\pi_1\Sigma_g, \mathrm{PSL}(2,\mathbb{R}))/\mathrm{PSL}(2,\mathbb{R})$, called the Teichmüller space $\mathcal{T}(\Sigma_g)$ of the surface. The interpretation of all points in $\mathcal{T}(\Sigma_g)$ as hyperbolic structures (as well as complex structures) on Σ_g yields a very rich understanding of this space, and a general problem consists in generalizing this viewpoint. If one considers $\mathrm{PSL}(2,\mathbb{C})$, or $\mathrm{PSL}(n,\mathbb{R})$ instead of $\mathrm{PSL}(2,\mathbb{R})$, the "prefered" representations are then the quasi-Fuchsian, or the

Hitchin representations of the surface group. On the other hand, if one considers non-Teichmüller representations in $\mathrm{PSL}(2,\mathbb{R})$, these can be related to geometric objects (such as branched hyperbolic structures on Σ_g, on anti-de Sitter structures on circle bundles over Σ_g), but this geometric interpretation is not canonical, and the corresponding moduli spaces are still not very well understood.

Closely related questions are to understand directly the topology of these representation spaces, and to understand the dynamics of the mapping class group of Σ_g on these spaces.

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Geometric group theory and the Farrell-Jones Conjecture

My current research has been focused on the interaction between geometric group theory and algebraic K-theory, in particular, I am working on the Farrell-Jones Conjecture. The Farrell-Jones Conjecture plays a very important role in geometric topology. It implies for example the Borel Conjecture and the Novikov Conjecture, which are considered as two of the most important unsolved problems in geometric topology nowdays. The conjecture also has deep connection with other conjectures in algebra, for example, Bass Conjecure, Kaplansky Conjecture.

My work has been concentrated on verifying the conjecture for certain classes of groups. This usually involves studying the groups with geometric group theory techniques, for example find a suitable geometric model that the groups act on. In more detail, together with my advisor F. T. Farrell, we proved in [1] and [3] that the Farrell-Jones Conjecture is true for the Baumslag-Solitar groups, which is a problem more than ten years old. Later, we generalized our method in [1], and proved the Farrell-Jones Conjecture is true for a large class of nearly crystallgraphic groups in [2]. Recently, I observed that the Farrell-Jones Conjecture is true for all elementary amenable groups with finite Hirsch lengths ([4]). In the future, I mainly plan to study two classes of groups, free by cyclic groups and groups acting on a CAT(0) space in a nice way.

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