Evolution of starshaped hypersurfaces by general curvature functions

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We investigate a fully nonlinear evolution equation for hypersurfaces in the Euclidean space. More precisely, let $f : \mathbb{R}^{n+1} \to \mathbb{R}$ $(n \geq 2)$ be a smooth function, and let $M_0 \subset \mathbb{R}^{n+1}$ be a starshaped hypersurface, which we suppose parametrized by a smooth immersion $X_0 : \mathbb{S}^n \to \mathbb{R}^{n+1}$. Then we consider the following evolution problem :

$$\begin{cases} \partial_t X(t,x) = \Big(K(\kappa_1, \cdots, \kappa_n) - f(X(t,x)) \Big) \nu(t,x) \\ X(0,x) = X_0(x) \ , \end{cases}$$

where $X : [0,T) \times \mathbb{S}^n \to \mathbb{R}^{n+1}$ such that, for each $t \in [0,T)$, $X(t,.)$ is the parametrization of a starshaped hypersurface M_t, $\nu(t,x)$ is the outer unit normal vector field at $X(t,x)$, and K is a given function of the principal curvatures $(\kappa_1, \cdots, \kappa_n)$ of M_t, which we suppose symmetric and homogeneous of degre $k \leq 0$.

We prove, under some assumptions on K and f, that the above evolution equation admits a global smooth solution defined on $[0, +\infty)$, and as $t \to +\infty$, it converges to a smooth map $X_\infty : \mathbb{S}^n \to \mathbb{R}^{n+1}$ parametrizing a starshaped hypersurface M_∞, whose principal curvatures satisfy the following prescribed curvature equation :

$$K(\kappa_1, \cdots, \kappa_n) = f(X).$$