Errata for the book "Homotopy of Operads and Grothendieck–Teichmüller Groups" *by Benoit Fresse*
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This list of errata will be regularly updated. Please send your comments to the author if you spot new mistakes (see mailing address at the end of the list).

Errata of Part 1. The algebraic Theory and its Topological Background

*** Page 169:** *The second displayed formula should be*

$$(\tau_k)_*(n_1,\dots,n_r) = (\tau_{k_i+n_i}\tau_{k_i+n_i-1}\cdots\tau_{k_i+1})\cdot(\tau_{k_i+n_i+1}\tau_{k_i+n_i}\cdots\tau_{k_i+2})\cdot\ldots \cdot(\tau_{k_i+n_i+n_{i+1}-1}\tau_{k_i+n_i+n_{i+1}-2}\cdots\tau_{k_i+n_{i+1}}),$$

where the indices of the braid generators τ_l in the last group of factors run from $l = k_i + n_{i+1}$ up to $l = k_i + n_i + n_{i+1} - 1$ (instead of up to $l = k_i + n_i + n_{i+1}$). In the picture of Figure I.5.7, these factors correspond to the crossings of the last strand of the ith block of the braid with the n_i strands of the $i+1$st block.

*** Page 299, line 5:** "(...) where the product ranges over all collections $\underline{i} = (i_2,\dots,i_s)$ and where (...)" *since the indices of such collections are numbered from 2 to s in the formula.*

*** Page 382, lines -9-8:** "(...) we easily check that we have the formulas $\phi(t_{12}) = \lambda t_{12}$ and $\phi(t_{23}) = f(t_{12},t_{23})^{-1}\cdot\lambda t_{23}\cdot f(t_{12},t_{23})$ (...)" *since one has to determine the image of the homomorphisms represented by the elements t_{12} and t_{23} (but not t_{13}) in order to compute $\phi(g(t_{12},t_{23})) = g(\phi(t_{12}),\phi(t_{23}))$.*

*** Page 504, lines 3-4:** *Insert the entry* "$\mathcal{H}opf\,\mathcal{C}at$: the category of Hopf categories (defined as the category of small categories enriched in counitary cocommutative coalgebras), §I.9.0.1" *and correct the entry* "$\mathcal{H}opf\,\mathcal{G}rd$" *on lines 3-4 to* "$\mathcal{H}opf\,\mathcal{G}rd$: the category of Hopf groupoids (defined as the category of Hopf categories equipped with an antipode operation), §I.9.0.2" *since a Hopf groupoid is a Hopf category and not a small category in the ordinary sense.*

*** Page 508:** *Add the following entries to the "Notation of operads" section:* "P_+: the unitary extension of a non-unitary operad, see §§I.1.1.19-1.1.20, and the unitary operad associated to an augmented Λ-operad, see §I.2.2"*;* "As_+: the unitary version of the associative operad, see §I.1.1.16, §I.1.2.8"*;* "Com_+: the unitary version of the commutative operad, see §I.1.1.16, §I.1.2.8"*.*

*** Page 522, second column, lines 6-8:** *This reference* "chord diagram, and the Malcev completion of the parenthesized braid operad, 363" *should be integrated in the entry* "chord diagram operad, and the Malcev completion of the parenthesized braid operad" *on lines 11-12. (Thus, there should be no general entry "chord diagram" in the index.)*

*** Page 522, second column, lines 20-21:** *This reference* "chord diagrams, algebras of, 348" *should be integrated in the entry* "chord diagram algebras" *on line 9. (Thus, there should be no entry called "chord diagrams" in the index.)*

Errata of Part 2. The Applications of (Rational) Homotopy Theory Methods

*** Page 131, lines 14-15:** "(…) see [38, §3], [119, §17] for standard references on this subject (…)" *since this is May's book on simplicial sets [119] and not May's book on iterated loop spaces [118] which is cited at this place.*

*** Pages 284-294, §II.9.2, section title:** *In the title of §II.9.2 (and at a few other places scattered throughout the book), the phrase "cochain graded dg-cooperads" is used for the category of cooperads in cochain graded dg-modules, but in the book, this category of cooperads is more usually referred to by the short name "cochain dg-cooperads" (as specified in the introduction of §II.9.2).*

*** Page 301, line -20:** "(…) of the components of our object $C^{\bullet}$." *with a full stop (instead of a comma) at the end of the sentence.*

*** Page 307, line -10:** "9.4.6. The conormalized cochain complex ~~of~~ as a totalization functor." *(Remove the word "of".)*

*** Page 465, lines -18-17:** "We refer to the textbooks [76, Lemma II.7.3] and [119, §7] (…)" *since this is again May's book on simplicial sets [119] and not May's book on iterated loop spaces [118] which is cited at this place.*

*** Page 545, lines 1-3:** *The composition formula on line 5 actually corresponds to a choice of generating elements $\mathbb{p}_r^s \in \Lambda^s(r)$ that differs from the one given on lines 1-3. Thus, the definition of lines 1-3 has to corrected in order to make this definition coherent with the composition formula of line 5. To fix things, one has just to replace the sentence* "We now have $\Lambda^s(r) = \mathbb{Q}\,\mathbb{p}_r^s$ (…)." *on lines 1-3 by the following:* "To define this operad Λ^s, we can equivalently take $\Lambda^s = \mathtt{End}_{\mathbb{Q}[s]}$ where we consider the endomorphism operad associated to the graded module $\mathbb{Q}[s]$. Indeed, we have a canonical isomorphism $\mathtt{Hom}(\mathbb{Q}[1]^{\otimes r}, \mathbb{Q}[1])^{\otimes s} \simeq \mathtt{Hom}(\mathbb{Q}[s]^{\otimes r}, \mathbb{Q}[s])$, yielded by the identity $\mathbb{Q}[1]^{\otimes s} = \mathbb{Q}[s]$ and the obvious shuffle operation on tensors, and we therefore have the relation $\Lambda(r)^{\otimes s} \simeq \mathtt{End}_{\mathbb{Q}[s]}(r)$, in each arity $r > 0$, when we take the s-fold tensor power of the operad Λ. For this operad $\Lambda^s = \mathtt{End}_{\mathbb{Q}[s]}$, we still have an obvious identity $\Lambda^s(r) = \mathbb{Q}\,\mathbb{p}_r^s$, but $\mathbb{p}_r^s$ now denotes an element of degree $s(1-r)$, which we assume to be given by the obvious homomorphism $\mathbb{Q}[s]^{\otimes r} = \mathbb{Q}[sr] \to \mathbb{Q}[s]$."
Indeed, if we make this choice for the elements $\mathbb{p}_r^s \in \Lambda^s(r)$, then we get the formula $\mathbb{p}_m^s \circ_k \mathbb{p}_n^s = (-1)^{s(k-1)(n-1)}\,\mathbb{p}_{m+n-1}^s$ in the endomorphism operad $\Lambda^s = \mathtt{End}_{\mathbb{Q}[s]}$. Thus, we do get the formula of line 5 in this case. In fact, if we write $\mathbb{Q}[s] = \mathbb{Q}\,\mathbb{e}_s$ and we take the image of the tensor product $\mathbb{p}_r^{\otimes s} \in \Lambda(r)^{\otimes s}$ under our shuffle isomorphism $\mathtt{Hom}(\mathbb{Q}[1]^{\otimes r}, \mathbb{Q}[1])^{\otimes s} \simeq \mathtt{Hom}(\mathbb{Q}[s]^{\otimes r}, \mathbb{Q}[s])$, then we get the formula $\mathbb{p}_r^{\otimes s}(\mathbb{e}_s^{\otimes r}) = (-1)^{\sigma}\,\mathbb{e}_s$, for a sign such that $\sigma = \frac{(r-1)(r-2)}{2}\frac{(s-1)(s-2)}{2}$. Thus, we eventually have the relation $\mathbb{p}_r^{\otimes s} = (-1)^{\sigma}\,\mathbb{p}_r^s$ when we assume that the elements $\mathbb{p}_r^s \in \Lambda^s(r)$ are given by the obvious map $\mathbb{p}_r^s(\mathbb{e}_s^{\otimes r}) = \mathbb{e}_s$ as in the above corrected definition.

*** Page 672, lines 3-4:** *Insert the entry* "$\mathcal{H}opf\mathcal{C}at$: the category of Hopf categories (defined as the category of small categories enriched in counitary cocommutative coalgebras), §I.9.0.1" *and correct the entry* "$\mathcal{H}opf\mathcal{G}rd$" *to* "$\mathcal{H}opf\mathcal{G}rd$: the category of Hopf groupoids (defined as the category of Hopf categories equipped with an antipode operation), §I.9.0.2" *since a Hopf groupoid is a Hopf category and not a small category in the ordinary sense.*

*** Page 676:** *Add the following entries to the "Notation of operads" section:* "P_+: the unitary extension of a non-unitary operad, see §§I.1.1.19-1.1.20, and the unitary operad associated to an augmented Λ-operad, see §I.2.2"*;* "As_+: the unitary version of the associative operad, see §I.1.1.16, §I.1.2.8"*;* "Com_+: the unitary version of the commutative operad, see §I.1.1.16, §I.1.2.8"*.*

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