# Lecture 2:
# Bipartite systems coupled to two baths:
# Time-evolution of Entanglement

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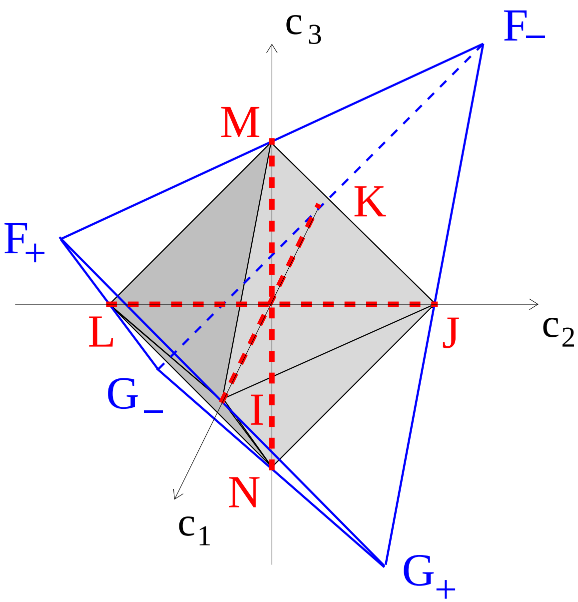
# Ex. of 2-qubit separable and classical states

◇ 2-qubit states with max. disordered marginals $\rho_{A/B} = \frac{1}{2}$ can be written (up to conjugation by a local unitary $U_A \otimes U_B$) as




$$\rho = \frac{1}{4}\Big(1 \otimes 1 + \sum_{m=1}^{3} c_m \sigma_m \otimes \sigma_m\Big)$$

$\vec{c} \in \mathcal{T}$ tetrahedron with vertices

$F_\pm = (\pm 1, \mp 1, 1)$, $G_\pm = (\pm 1, \pm 1, -1)$

◇ $\rho$ separable $\Leftrightarrow$ $\vec{c} \in$ octahedron $IJKLMN$

◇ Quantum discords [Luo, PRA '08] [DS & Orszag, '13]

$$\delta_A(\rho) = \sum_{\nu=0}^{3} p_\nu \ln p_\nu + \ln 4 - \frac{1-|c|}{2}\ln(1-|c|) - \frac{1+|c|}{2}\ln(1+|c|)$$

$$D_A(\rho) = 2\Big(1 - \sqrt{\frac{1+b_+ + b_-}{2}}\Big)\ ,\ b_\pm = \frac{1}{2}\sqrt{(1\pm c_1)^2 - (c_2 \mp c_3)^2}$$

if $|c| = \max_m |c_m| = |c_1|$ (else circular permut. of $(c_1, c_2, c_3)$)

$p_\nu =$ euclidean distance of the origin $O$ to the faces of $\mathcal{T}$

# Outlines

- Evolution of the concurrence and quantum discord for the 2 spin-boson model
- Average concurrence for quantum trajectories
- Protecting entanglement with quantum trajectories
- Qubits coupled to a common bath
- Conclusions & Perspectives

**Joint work with:** Sylvain Vogelsberger

# The 2 spin-boson model

*[Yu and Eberly ('04)]; Merkli et al. ('10); Merkli ('11), ...]*

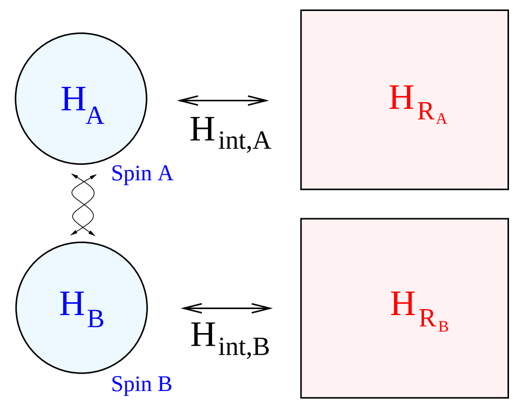
- Consider two spins $A$ and $B$ coupled to two independent free-boson reservoirs $R_A$ and $R_B$. There are no interactions between $A$ & $B$. The total Hamiltonian is:

$$H_{\text{tot}} = H_A + H_B + H_{R_A} + H_{R_B} + \lambda H_{\text{int}}$$

$$H_A = \omega_A \sigma_z^A \ , \ H_B = \omega_B \sigma_z^B$$

$$H_{R_A} = \sum_k \mu_k a_k^\dagger a_k$$

$$H_{R_B} = \sum_k \nu_k b_k^\dagger b_k$$




- Initial state: $\rho_{\text{tot}}(0) = \underbrace{\rho_{AB}(0)}_{\text{ENTANGLED}} \otimes \rho_{R_A} \otimes \rho_{R_B}$

- **MODEL 1:** spin-boson coupling given by Jaynes-Cummings:

$$H_{\text{int}} = \sum_k (g_k \sigma_+^A \otimes a_k + f_k \sigma_+^B \otimes b_k + \text{h.c.})$$

# The 2 spin-boson model (2)

- $\rho_{R_k}$ Gibbs states with inverse temperatures $\beta_k < \infty$

  $\Rightarrow$ the 2-spin state converges at large times to

  $$\rho_{AB}(\infty) = Z^{-1} e^{-\beta_A \omega_A \sigma_z^A} \otimes e^{-\beta_B \omega_B \sigma_z^B} \text{ product state}$$

- $\rho_{AB}(\infty)$ is in the *interior of the set of separable states* $\mathcal{S}$.

  **Reason:** $\rho_{AB} \in \partial\mathcal{S} \;\Leftrightarrow\; \rho_{AB}^{T_B}$ has at least one zero eigenvalue

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  (by the Peres-Horodecki criterium).
- By continuity of $t \mapsto \rho_{AB}(t)$, the 2-spin state
  $$\rho_{AB}(t) = \operatorname*{tr}_{R_A,R_B}\left(e^{-itH_{\rm tot}} \rho_{\rm tot}(0) e^{itH_{\rm tot}}\right)$$
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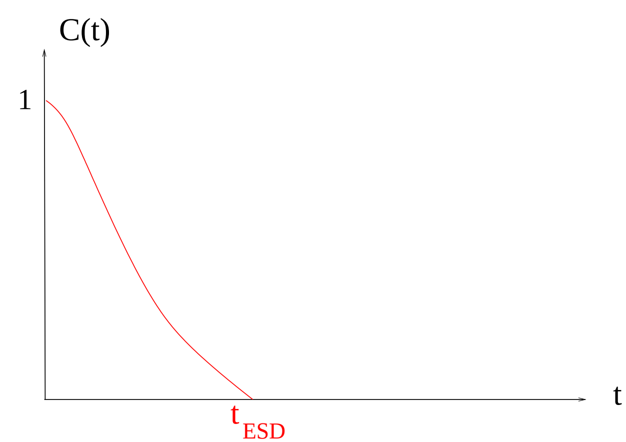
  $\hookrightarrow$ can be checked by computing the concurrence of $\rho_{AB}(t)$ in the weak coupling limit *[Yu and Eberly ('04),...]*, or by using the resonance perturbation theory *[Merkli et al. ('10)]*.



# 2 spin-boson model: pure phase dephasing

- **MODEL 2:**

$$H_{\text{int}} = \sum_k \big( g_k \sigma_z^A \otimes (a_k + a_k^\dagger) + f_k \sigma_z^B \otimes (b_k + b_k^\dagger) \big)$$

Weak coupling limit (but not necessary)

$$\frac{d}{dt}\rho_{AB} = \gamma_z^A \big( \sigma_z^A \rho_{AB}(t) \sigma_z^A - \rho_{AB}(t) \big) + (A \leftrightarrow B)$$

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Initial 2-spin state with maximally mixed marginals

$$\rho_{AB}(0) = \frac{1}{4}\Big(1 \otimes 1 + \sum_{m=1}^{3} c_m \sigma_m \otimes \sigma_m\Big) \ , \ \vec{c} \in \mathcal{T}$$

$\hookrightarrow$ remains of this form at all times $t \geq 0$, with

$$c_1(t) = e^{-2\gamma t} c_1 \ , \ c_2(t) = e^{-2\gamma t} c_2 \ \text{ and } \ c_3(t) = c_3$$

$\gamma = \gamma_z^A + \gamma_z^B$.

# 2 spin-boson model: pure phase dephasing

- **Concurrence:**

$$C[\rho_{AB}(t)] = \frac{1}{2}\max\{0, |c_1 \pm c_2|e^{-2\gamma t} - 1 \mp c_3\}$$

  Initial state entangled $\Leftrightarrow |c_1 \pm c_2| > 1 \pm c_3$ ($+$ or $-$).

- For $c_3 = \mp 1$ (e.g. initial Bell state $|\psi_{AB}\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$),
  $C[\rho_{AB}(t)] = |c_1 \pm c_2|e^{-2\gamma t}$ **vanishes asymptotically**.

  Otherwise entanglement is lost after a **finite time**

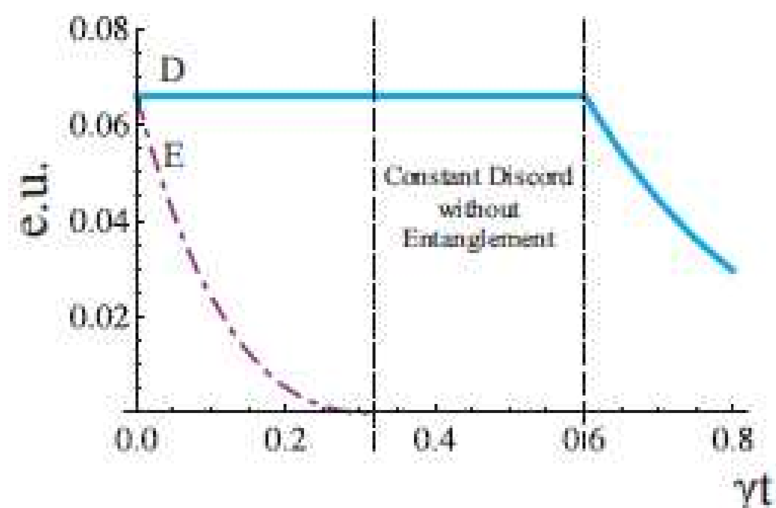
$$t_{ESD} = \frac{1}{2\gamma}\ln\Big(\max\{\frac{|c_1 \pm c_2|}{1 \pm c_3}\}\Big).$$

- For $c_1 = \pm 1$, $c_2 = \mp c_3$ with $|c_3| < 1$,

$$\delta_A[\rho_{AB}(t)] = \text{const.} \quad \text{for}$$

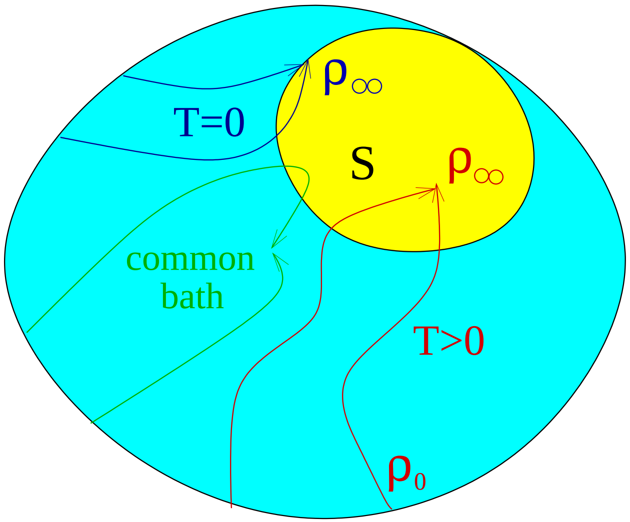
$$0 \le t \le -\frac{\ln|c_3|}{2\gamma}.$$

  *[Mazzola, Piilo & Maniscalco ('10)]*



# Entanglement sudden death and birth

ENTANGLEMENT TYPICALLY DISAPPEARS BEFORE COHERENCES ARE LOST!




It can disappear after a finite time

- *always the case if the qubits relax to a Gibbs state $\rho_\infty$ at positive temperature*
- *otherwise depends on the initial state.*

*[Diosi '03], [Dodd & Halliwell PRA 69 ('04)], [Yu et Eberly PRL 93 ('04)]*

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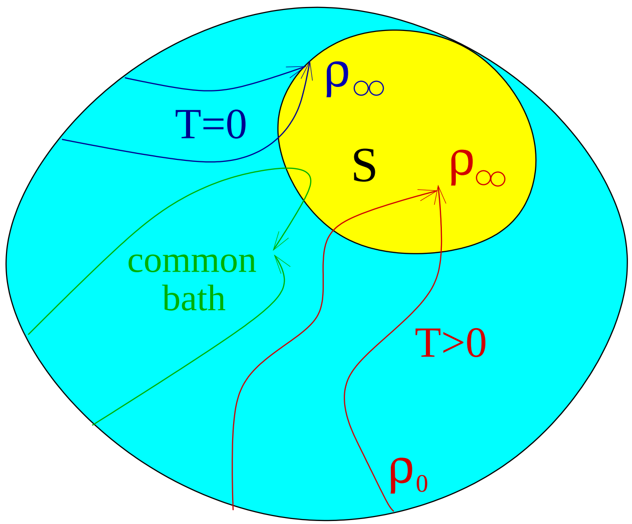


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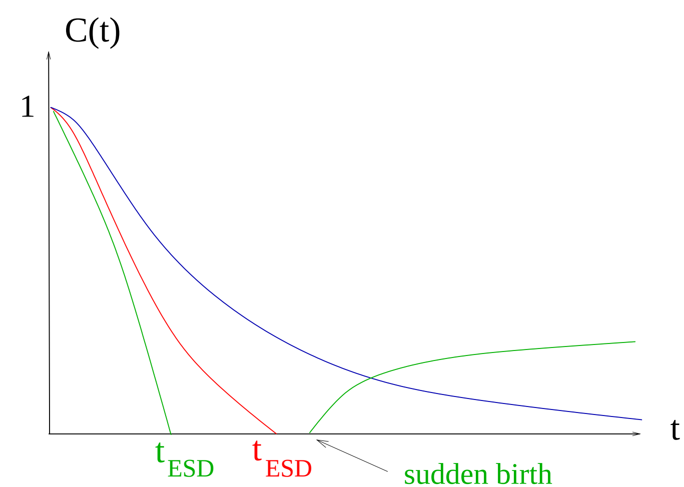
If the two qubits are coupled to a **common bath**, entanglement can also suddently reappear

$\rightsquigarrow$ *due to effective (bath-mediated) qubit interaction creating entanglement*

*[Ficek & Tanás PRA 74 ('06)], [Hernandez & Orszag PRA 78 ('08)], [Mazzola et al. PRA ('09)]*



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# Quantum trajectories

As a result of continuous measurements on the environment, the bipartite system remains in a pure state $|\psi(t)\rangle$ at all times $t > 0$

$$t \in \mathbb{R}_+ \mapsto |\psi(t)\rangle \quad \text{quantum trajectory}$$

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In general this decomposition is NOT THE OPTIMAL one,

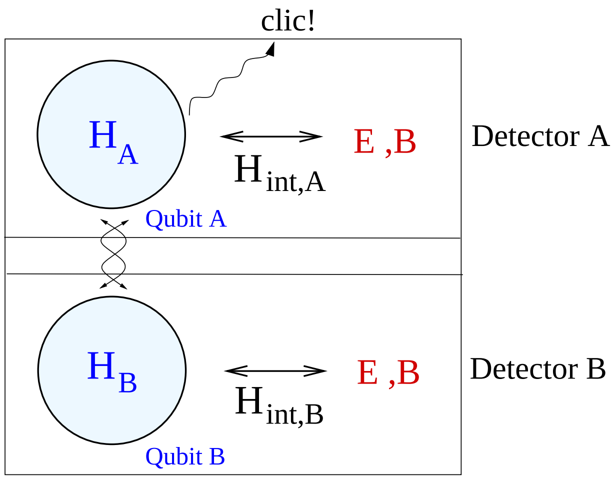
$$\overline{E_{\psi(t)}} \geq E_{\rho(t)}$$

*[Nha & Carmichael PRL 98 ('04)].*

But for specific models, one can find measurement schemes with $\overline{C_{\psi(t)}} = C_{\rho(t)}\ \forall\, t \geq 0$ with $C =$ Wootters concurrence for 2 qubits

*[Carvalho et al. PRL 98 ('07), Viviescas et al. ('10)].*

# Photon counting




Two 2-level atoms (qubits) initially in state $|\psi\rangle = \sum_{s,s'=0,1} c_{ss'}|s\rangle|s'\rangle$ are coupled to independent modes of the electromagnetic field initially in the vacuum.

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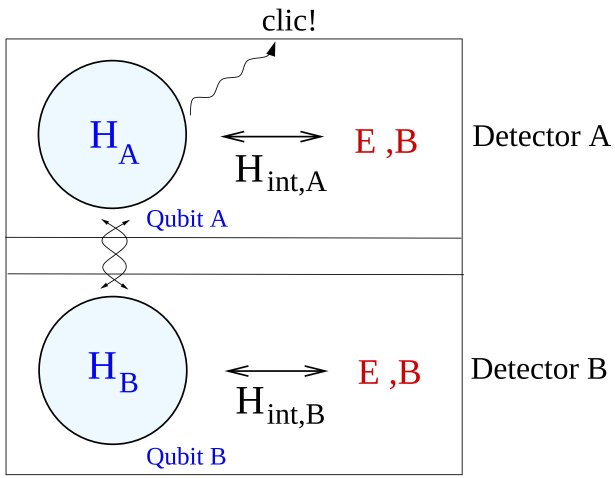


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- If $D_i$ detects a photon between $t$ and $t + \mathrm{d}t$, the qubits suffer a quantum jump [occurs with proba. $\gamma_i \|\sigma_-^i |\psi(t)\rangle\|^2 \mathrm{d}t$]

$$|\psi(t)\rangle \longrightarrow \sigma_-^i |\psi(t)\rangle = |0\rangle_i \otimes |\phi(t)\rangle \quad \rightsquigarrow \text{separable.}$$

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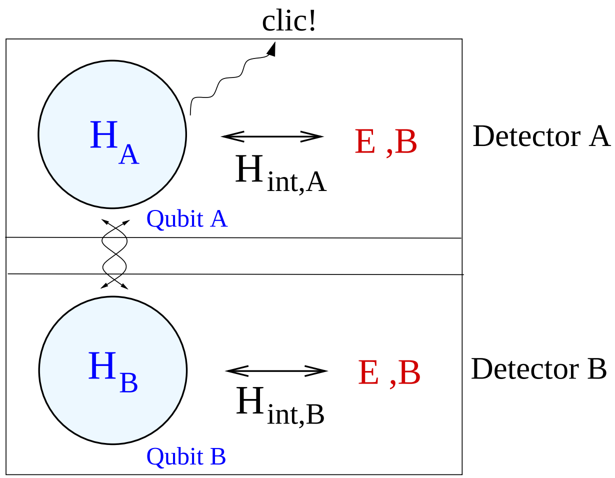


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- If no click occurs between $t_0$ and $t$ [proba. $\|e^{-itH_{\text{eff}}}|\psi(t_0)\rangle\|^2$]

$$|\psi(t)\rangle = \frac{e^{-i(t-t_0)H_{\text{eff}}}|\psi(t_0)\rangle}{\|e^{-itH_{\text{eff}}}|\psi(t_0)\rangle\|}\ ,\quad H_{\text{eff}} = H_0 - \frac{i}{2}\sum_{i=A,B}\gamma_i\,\sigma_+^i\sigma_-^i .$$

# Photon counting (2)

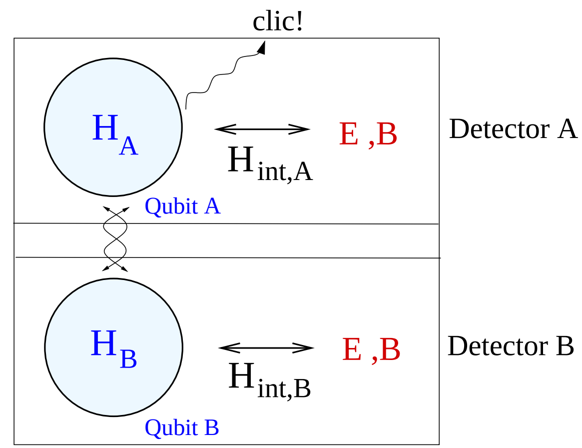
Concurrence:

$$C_{\psi(t)} = |\langle \psi(t)|\sigma_y \otimes \sigma_y T|\psi(t)\rangle|$$

$T =$ complex conjugation op.

$\sigma_y =$ Pauli matrix

$\hookrightarrow E_{\psi(t)} = h(C_{\psi(t)})$, $h$ convex $\nearrow$




- Trajectories with 1 or more jumps between $0$ and $t$ have a concurrence $C_{\psi(t)} = 0$ (since $|\psi(t)\rangle$ separable after 1 jump).
- If no jump occurs between $0$ and $t$, one finds for $H_0 = 0$:

$$C_{\text{no jump}}(t) = \mathcal{N}_t^{-2}\, C_0\, e^{-(\gamma_A+\gamma_B)t} \quad \text{with} \quad \mathcal{N}_t = \| e^{-itH_{\text{eff}}}|\psi\rangle \|.$$

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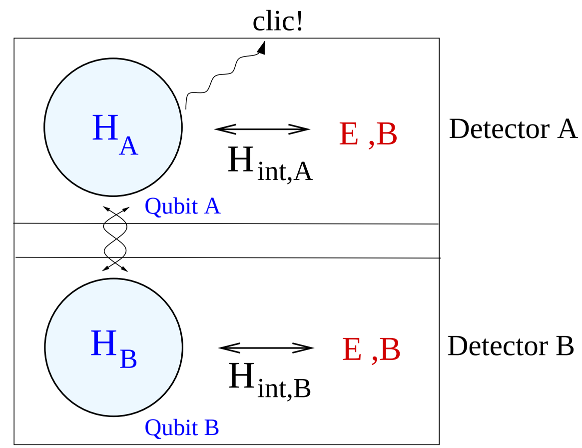


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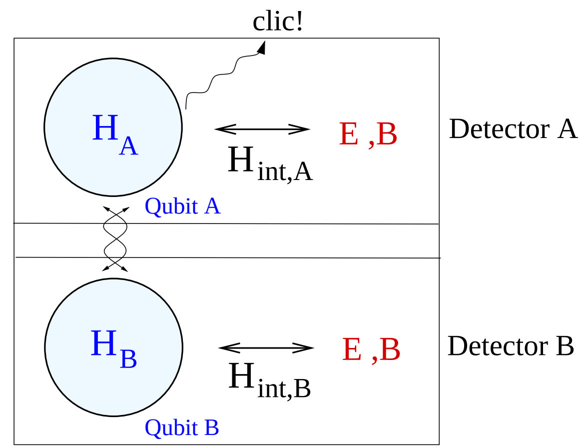


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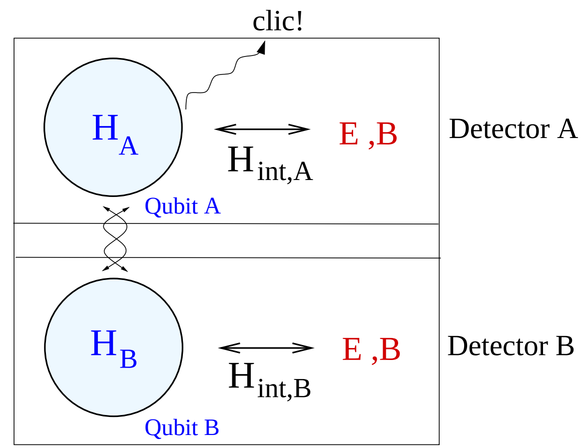


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$\hookrightarrow \overline{C_{\psi(t)}}$ vanishes asymptotically $\Rightarrow$ **sudden death of entanglement never occurs for quantum trajectories!**

# General quantum jump dynamics

Consider 2 noninteracting qubits coupled to *independent baths* monitored by means of *local measurements*

$\Rightarrow$ the jump operators $J = J^A \otimes 1$ or $1_A \otimes J_B$ are *local*.

- *The no-jump trajectories have a non-vanishing concurrence $C_{\rm nj}(t) > 0$ at all finite times* (if $C_0 > 0$).

  **Proof:** assume the contrary, i.e. $|\psi_{\text{nj}}(t)\rangle$ separable, then $|\psi(0)\rangle \propto e^{itH_{\rm eff}}|\psi_{\text{nj}}(t)\rangle$ would be separable since $e^{itH_{\rm eff}}$ is a tensor product of two local operators acting on each qubits.

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- *The average concurrence over all trajectories is*

$$\overline{C_{\psi(t)}} = C_0\, e^{-\kappa t}$$

  *where* $\kappa \geq 0$ *depends on the measurement scheme only (but not on initial state).* *[Vogelsberger & D.S, PRA ('10)]*

**Note:** $\overline{E_{\psi(t)}} \geq h(\overline{C_{\psi(t)}})$ by convexity of $h$.

# Quantum state diffusion

- The result $\overline{C_{\psi(t)}} = C_0\, e^{-\kappa t}$ is not only true for quantum jump dynamics but also for quantum state diffusion, e.g. for trajectories given by the stochastic Schrödinger equation

$$
\begin{aligned}
|\mathrm{d}\psi\rangle \;=\; & \Big[(-iH_0 - K)\mathrm{d}t + \sum_{J \text{ local}} \gamma_J \Big(\Re\langle J\rangle_\psi\, J - \frac{1}{2}(\Re\langle J\rangle_\psi)^2\Big)\mathrm{d}t \\
& + \sum \sqrt{\gamma_J}\big(J - \Re\langle J\rangle_\psi\big)\mathrm{d}w\Big]|\psi\rangle
\end{aligned}
$$

  which describes **homodyne detection**.

- The disentanglement rates $\kappa$ are different for photon-counting, homodyne, and heterodyne detections:

$$
\begin{aligned}
\kappa_{\mathrm{QJ}} &= \frac{1}{2}\sum_J \gamma_J \Big(\mathrm{tr}(J^\dagger J) - 2|\det(J)|\Big) \\
\kappa_{\mathrm{ho}} &= \frac{1}{2}\sum_J \gamma_J \Big(\mathrm{tr}(J^\dagger J) - 2\Re\det(J) - (\Im\,\mathrm{tr}(J))^2\Big) \\
\kappa_{\mathrm{het}} &= \frac{1}{2}\sum_J \gamma_J \Big(\mathrm{tr}(J^\dagger J) - \frac{1}{2}|\,\mathrm{tr}(J)|\Big).
\end{aligned}
$$

  Adjusting the laser phases $J \to e^{-i\theta} J$ yields $\kappa_{\mathrm{ho}} \leq \kappa_{\mathrm{QJ}}, \kappa_{\mathrm{het}}$.

# Discussion

It is **not possible** to have $\overline{C_{\psi(t)}} = C_{\rho(t)}$ if one measures locally the independent environments of the qubits (since $C_{\rho(t)}$ may vanish at a finite time $t_{\rm ESD}$, whereas $\overline{C_{\psi(t)}} > 0 \ \ \forall\, t$).

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- $\ast$ This raises the question: **is ESD observable?**

  *[Almeida et al., Science 316 ('07)].* $\longrightarrow$ simulation of master eq.

  *[Viviescas et al., arXiv:1006.1452].* $\longrightarrow$ YES with some nonlocal measurements $\Rightarrow$ require additional quantum channels...

- $\ast$ For $A$-$B$ entanglement, "ignoring" the environment state is not the same as measuring it without reading the results.

  *[Mascararenhas et al., arXiv:1006.1233].*

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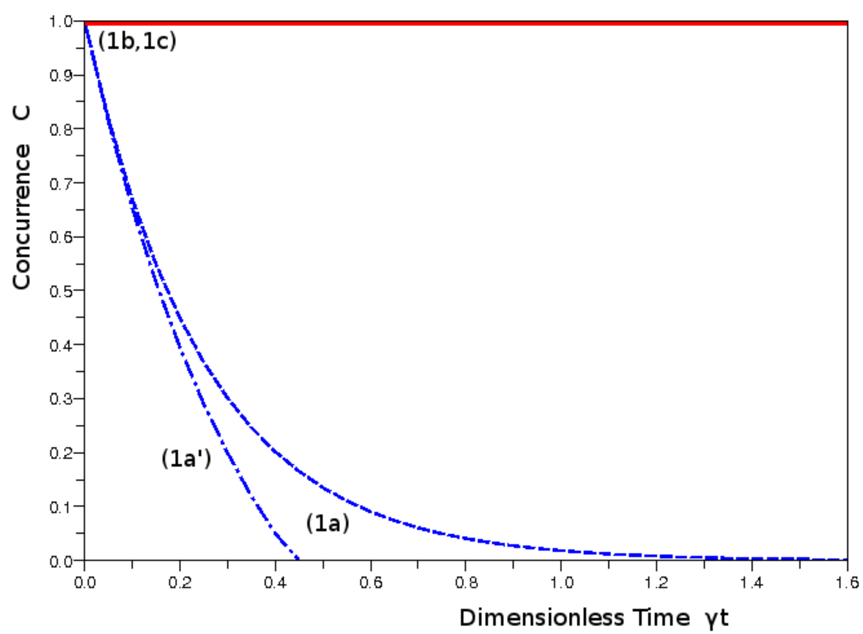
# Entanglement protection

One may use the continuous monitoring by the measurements to protect the qubits against disentanglement.

- For ex., for pure phase dephasing ($J^i = \mathbf{u}_i \cdot \sigma^i$, $i = A, B$), $\kappa_{\rm QJ} = \kappa_{\rm ho} = \kappa_{\rm het} = 0$ so that $\overline{C_{\psi(t)}} = C_0 = \text{const.}$




Bell initial state

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + e^{-i\varphi}|\downarrow\downarrow\rangle)$$

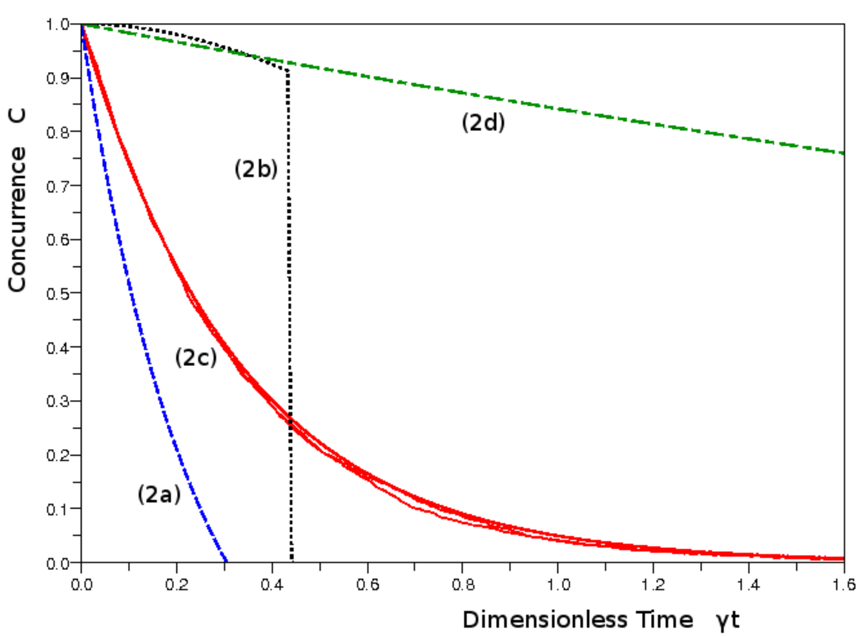
$C_0 = 1 \Rightarrow C_{\psi(t)} = 1$ for all quantum trajectories and all times

$\hookrightarrow$ perfect entanglement protection!

# Entanglement protection (2)

One may use the continuous monitoring by the measurements to protect the qubits against disentanglement.

- For ex., for pure phase dephasing ($J^i = \mathbf{u}_i \cdot \sigma^i$, $i = A, B$), $\kappa_{\text{QJ}} = \kappa_{\text{ho}} = \kappa_{\text{het}} = 0$ so that $\overline{C_{\psi(t)}} = C_0 = \text{const.}$
- For two baths at inv. temper. $\beta_i < \infty$, the smallest rate is $\kappa_{\text{QJ}} = \sum_{i=A,B} \gamma_+^i (e^{\beta_i \omega_0/2} - 1)^2$ (jump op. $J \propto \sqrt{\gamma_-^i}\sigma_-^i + \sqrt{\gamma_+^i}\sigma_+^i$)




Bell initial state

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle - i|\downarrow\downarrow\rangle)$$

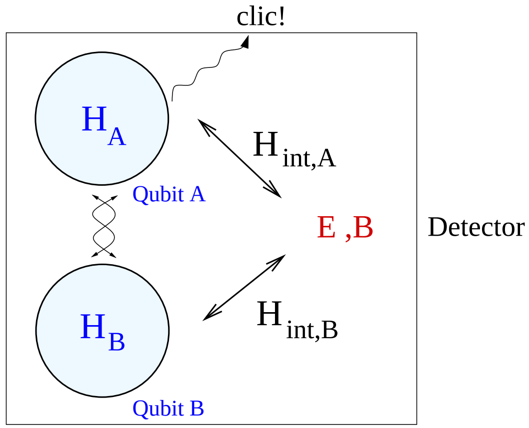
$$\overline{C_{\psi(t)}} = e^{-\kappa t}$$

$\hookrightarrow$ perfect entanglement protection only possible at infinite temperature!

# Outlines

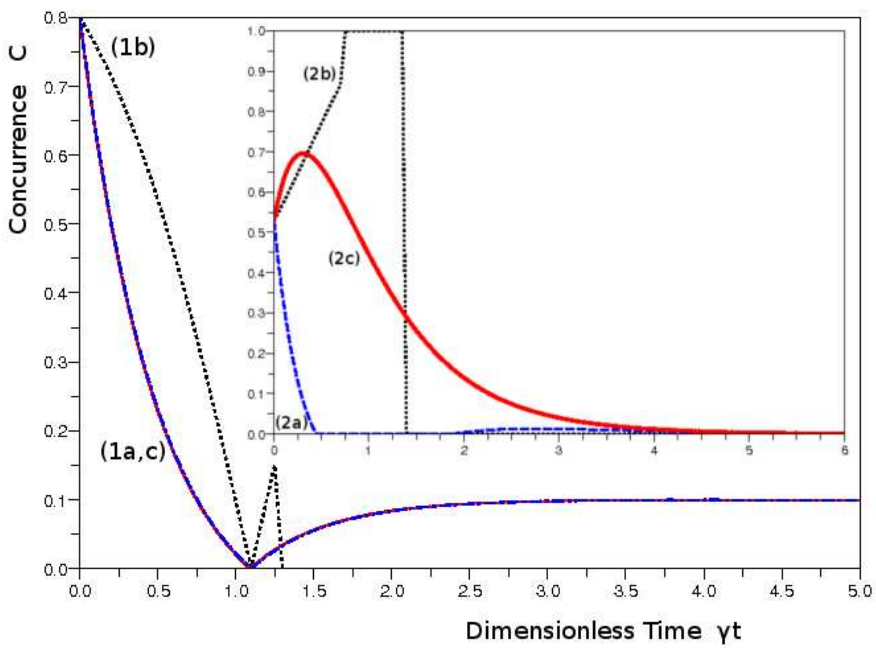
- Evolution of the concurrence and quantum discord for the 2 spin-boson model
- Average concurrence for quantum trajectories
- Protecting entanglement with quantum trajectories
- Qubits coupled to a common bath

# Qubits coupled to a common bath




Two 2-level atoms (qubits) initially in state $|\psi\rangle = \sum_{s,s'=0,1} c_{ss'}|s\rangle|s'\rangle$ are coupled to the **same** modes of the electromagnetic field initially in the vacuum.

$$\overline{C_{\psi(t)}} = \frac{1}{2}\left|c_-^2 - c_+^2 e^{-2\gamma t} + 4c_{11}c_{00}\, e^{-\gamma t}\right| + 2|c_{11}|^2\gamma t\, e^{-2\gamma t}$$




with $c_\pm = c_{11} \pm c_{00}$.

- If $c_{11} = 0$ then $\overline{C_{\psi(t)}} = C_{\rho(t)}$.
- If $c_{11} > 0$ then $\overline{C_{\psi(t)}}$ increases at small times.

# Outlines

- Evolution of the concurrence and quantum discord for the 2 spin-boson model
- Average concurrence for quantum trajectories
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- Conclusions & Perspectives

# Conclusions & Perspectives

- The mean concurrence $\overline{C(t)}$ of two qubits coupled to **independent baths** monitored by continuous **local measurements** decays exponentially with a rate depending on the measurement scheme only.

- Measuring the baths helps to protect entanglement, sometimes perfectly!

- For two qubits coupled to a **common bath**, the time behavior of the mean concurrence depends strongly on the initial state. One may have $\overline{C(t)} = C_{\rho(t)}$.

**Open problems:** non-Markov unravelings, multipartite systems,...