

Arithmétique des corps et géométrie diophantienne Field Arithmetic and Diophantine Geometry

11-13 Décembre 2013

Laboratoire Paul Painlevé, Université Lille 1

Lecture room: salle de réunions (bâtiment M2)

Contact: Pierre Dèbes (Pierre.Debes@univ-lille1.fr)

Description: The goal of the meeting is to bring together experts from the two parts of the title --- Field Arithmetic and Diophantine Geometry --- and to encourage interaction between them. We will focus on topics at the interplay of the two parts. This meeting is also the first event of the special semester "Arithmetic and algebraic geometry, applications to physics" organized by the laboratory Paul Painlevé and the LabeX CEMPI in 2014.

Speakers:

Francesco Amoroso - Caen
Bruno Anglès – Caen
Lior Bary - Soroker - Tel Aviv (Israël)
Sara Checcoli - Grenoble
Sinnou David - Paris
Pierre Dèbes - Lille
Bruno Deschamps - Le Mans
Arno Fehm - Konstanz (Allemagne)
Jochen Koenigsmann - Oxford (Royaume-Uni)
François Legrand - Lille

PROGRAM:

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WEDNESDAY, DECEMBER 11:

- 9h: welcome coffee
- 9h30-10h30: Bruno Anglès (Caen)
"On the special values of certain non-archimedean L-functions in positive characteristic"
- 10h30-11h: coffee break
- 11h-12h: Jochen Koenigsmann (Oxford)
TBA
- 12h30: Lunch
- 14h30-15h30: Sara Checcoli (Grenoble)
"On the properties of Northcott and Bogomolov"
- 15h30-16h: coffee break
- 16h-17h: Bruno Deschamps (Le Mans)
"Conjecture de Shafarevich, corps abyssaux et nombres premiers de Fermat"

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THURSDAY, DECEMBER 12

- 9h30-10h30: Arno Fehm (Konstanz)
"Varieties of Hilbert type"
- 10h30-11h: coffee break
- 11h-12h: Sinnou David (Paris)
TBA
- 14h30-15h30: Lior Bary-Soroker (Tel Aviv)
"Ramification under specializations and the Bateman-Horn conjecture"
- 15h30-16h: coffee break
- 16h-17h: François Legrand (Lille)
"Specializations of regular Galois extensions of $\mathbb{Q}(T)$ with specified local behavior"

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FRIDAY, DECEMBER 13

- 9h30-10h30: Pierre Dèbes (Lille)
"On the Malle conjecture with local conditions"

- 10h30-11h: coffee break

- 11h-12h: Francesco Amoroso (Caen)
"Lower bounds for the height in some infinite extensions"

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Participants

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Abstracts:

Francesco AMOROSO (Caen)

"Lower bounds for the height in some infinite extensions"

Abstract: In 2001 Bombieri and Zannier introduced the notion of fields having the so-called Bogomolov property (where the height is bounded below outside torsion points), denoted Property (B). In this talk we discuss some recent results on this subject".

Bruno ANGLES (Caen)

"On the special values of certain non-archimedean L-functions in positive characteristic"

Abstract: A major theme in the arithmetic theory of function fields over finite fields is to "understand" the arithmetic

of the special values of D. Goss L-functions introduced by Goss in the eighties (they are an analogue in positive characteristic of Artin L-functions). In 2012, F. Pellarin has introduced a new class of L-functions, this class

contains the class of D. Goss abelian L -functions by specialization. In this talk, we will give a survey on recent

results on the arithmetic of special values of Pellarin/Goss L-functions.

Lior BARY-SOROKER (Tel Aviv)

"Ramification under specializations and the Bateman-Horn conjecture"

Abstract: I will talk about a new attack on the minimal ramification problem: Let G be a nontrivial finite group. The inverse Galois problem asks whether there exists a Galois extension N of the rational numbers \mathbb{Q} with group G .

A classical variant of this problem, the "minimal ramification problem", asks to calculate $m(G)$ -- the minimal number $m(G)$ of prime numbers that ramify in N , where N varies over the G -extension of \mathbb{Q} . All previous attacks on the minimal ramification are based on number theoretical approaches, either by Galois cohomology and local global principles or by Galois representations; hence are restricted to the family of solvable groups, and certain matrix groups respectively. We develop a new machinery to bound ramifications in specializations, which is applicable to Galois extension of $\mathbb{Q}(t)$ one gets by rigidity methods. Recall that the Bateman-Horn conjecture says that for non-associate irreducible integral polynomials $f_1(X), \dots, f_r(X)$ with positive leading coefficients and with no local obstructions (i.e. such that no p divides all values of the product $f_1(n)\dots f_r(n)$, n runs over the integers) the number of $1 < a < x$ such that $f_1(a), \dots, f_r(a)$ are all primes is about $C x / \log^r(x)$ for large x . Here $C = C(f_1, \dots, f_r)$ is a positive constant. Combining the machinery we develop together with the Bateman-Horn conjecture or with the partial results toward the Bateman-Horn conjecture one gets by sieve methods we get surprising new results even for groups like S_n : S_n is realizable over \mathbb{Q} with a "bounded" number of ramified primes; in fact $m(S_n) = 1$ conditionally on BH and $m(S_n) < 19$ unconditionally. This talk is based on a joint work with Tomer Schlank

Sara CHECCOLI (Grenoble)

``On the properties of Northcott and Bogomolov"

Abstract: A set of algebraic numbers has the Northcott property (N) if it contains finitely many elements of bounded logarithmic Weil height; it has the Bogomolov property (B) if its elements have height either zero or bounded from below by an absolute positive constant. These properties, introduced by Bombieri and Zannier, were recently formulated for groups. Following Amoroso, David and Zannier, we say that a profinite group G has property (B) (resp. (N)) if for any number field K and for any Galois extension L/K of Galois group G , the field L satisfies (B) (resp. (N)). It is a natural and quite studied problem to investigate which subfields of $\overline{\mathbb{Q}}$ and which groups have property (N) or (B). After a brief survey, I will discuss some results obtained in collaboration with M. Widmer and some open questions.

Pierre DÈBES (Lille)

``On the Malle conjecture with local conditions"

Abstract. We show that for G in a large class of finite groups, the number of Galois extensions E/\mathbb{Q} of group G and discriminant $|\Delta_E| \leq y$ grows like a positive power of y . The extensions E/\mathbb{Q} we produce can further be prescribed any unramified local behavior at each suitably large prime $p \leq \log(y)^\delta$, for some $\delta \in]0, 1]$. This result is a step toward the Malle conjecture on the number of Galois extensions of given group and bounded discriminant. Due to the local conditions it also has implications toward the Grunwald problem and the Tchebotarev theorem.

Bruno DESCHAMPS (Le Mans)

``Conjecture de Shafarevich, corps abyssaux et nombres premiers de Fermat"

Abstract. The absolute Galois group of the field of rational numbers, $\text{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$, is still a not very well-known object which has some monstrous aspects. For fifty years, many results and conjectures came to support the idea that, upgrading a little the base field \mathbb{Q} could get much more natural absolute Galois groups. One of the most famous conjectures in this way is the one made by Shafarevich in 1964 which provides that the absolute Galois group of the abelian closure of the field of rationals $\text{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}^{\text{ab}})$ is a free profinite group. In this talk, after comparing this conjecture to more modern ones, we will try to explore the projective subfields of \mathbb{Q}^{ab} and examine the optimality of the Shafarevich conjecture. This approach leads to the question of the existence of "abyssal" extensions and its surprising connection with some famous questions in analytic number theory, such as the problem of the finiteness of the set of prime Fermat numbers.

Arno FEHM (Konstanz)

``Varieties of Hilbert type"

Abstract: A K -variety X is said to be of Hilbert type if the set of rational points $X(K)$ is not thin in the sense of Serre.

I will speak about products of varieties of Hilbert type, algebraic groups of Hilbert type, homogeneous spaces of Hilbert type, and varieties of Hilbert type under base change.

François LEGRAND (Lille)

``Specializations of regular Galois extensions of $\mathbb{Q}(T)$ with specified local behavior''

Abstract: ``The main topic of the talk is the construction, by specializing regular Galois extensions of $\mathbb{Q}(T)$ with group G , of Galois extensions of \mathbb{Q} with group G and specified local behavior (ramified or unramified) at finitely many given primes. In a first part, we will give the main result of the talk which has a ramified conclusion. In a second part, we will conjoin it with previous results which have unramified conclusions. If time permits, we will give an application to the construction of regular Galois extensions $E/\mathbb{Q}(T)$ with group G which satisfy the following property : at least one Galois extension F/\mathbb{Q} with group G is not a specialization of $E/\mathbb{Q}(T)$.''
