INTAS MEETING OF AMBLETEUSE

Abstracts of the talks

Alexander Aptekarev, Keldysh Institut, Moscow (Russia)

Title:" On a discrete entropy of ortogonal polynomials"

Abstract:

We shall discuss geometrical meaning of the discrete entropy of the eigenvector basis and particularly eigenvectors formed by orthogonal polynomials. The we present a nice new formula for the discrete entropy of Tchebyshev polynomials.

It is joint work with J.S.Dehesa, A.Martinez-Finkelstein and R.Yanez.

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Bernd Beckermann, University of Lille (France)

Title: "Smoothing the Gibbs phenomenon by Fourier-Padé techniques"

Abstract:

Partial Fourier sums of real-valued 2π-periodic functions $f(t) = \Re(\sum_{j=0}^{\infty} c_j e^{ijt})$ with jump discontinuities are known to converge slowly, not only close to the jump. Motivated by applications in spectral methods for PDE, several authors (Driscoll & Fornberg '01, Brezinski '02, Kaber & Maday '05) proposed recently to use Fourier-Padé approximants to overcome this Gibbs phenomenon of slow convergence. Following the book of Baker & Graves-Morris, one may imagine three different approximants $R = P/Q$ of f with P, Q trigonometric polynomials of degree at most $n + k$, and n, respectively: for the first (Wynn '67, Gragg-Johnson '74) we form the real part of the $[n + k|k]$ Padé approximant of $g(z) = \sum_{j=0}^{\infty} c_j z^j$ at $z = e^{it}$. For the second and third we follow the classical approach of rational approximants of orthogonal series and ask that the Fourier expansion of $fQ - P$, or of $f - P/Q$, contains only terms of order greater than $n + 2k$.

Brezinski showed numerically that the first approximants, which can be very efficiently evaluated by means of the epsilon algorithm, give raise to a spectacular acceleration of convergence, at least for t not too close to the jump. However, so far the only error estimate for the simple test function $f(t) = \text{sign}(\cos(t))$ and the case of columns (i.e., fixed k and $n \to \infty$) has been obtained by Kaber & Maday. It turns out that a much larger class of test functions with jumps (or with higher order derivative having a jump) is covered by the family of Stieltjes functions

$$g(z) = G^{(\alpha,\beta)}(z) = {}_2F_1\left(\begin{matrix} \alpha+1, 1 \\ \alpha+\beta+2 \end{matrix} \,\middle|\, z \right) = \sum_{j=0}^{\infty} \frac{(\alpha+1)_j}{(\alpha+\beta+2)_j} z^j, \quad \alpha, \beta \geq 0.$$

In this talk we first show that the first and the third Fourier-Padé approximant mentioned above are identical provided that the $[n+k|k]$ Padé approximant of g has no poles in the closed unit disk. Hence the theory of the Gaussian continued fraction gives (geometric) convergence of diagonal non-linear Padé-Fourier approximants for the above class of test functions. We then concentrate on column convergence (i.e., fixed k and $n \to \infty$) and obtain for the first approximants the rate of convergence $\mathcal{O}(n^{-2k-1})$ uniformly on compact subsets of $(0, 2\pi)$ for general Stieltjes functions g, and of $\mathcal{O}(n^{-2k-1-\beta})$ for the particular Stieltjes functions $G^{(\alpha,\beta)}$ mentioned above. Finally we present a Montessus de Ballore type theorem for perturbations of $G^{(\alpha,\beta)}(z)$, by showing that the above rate of convergence $\mathcal{O}(n^{-2k-1-\beta})$ is also valid provided that

$$f(t) - \Re(\sum_{j=0}^{\infty} \frac{(\alpha+1)_j}{(\alpha+\beta+2)_j} e^{ijt}) \in \mathcal{C}^{2k+1+\beta}.$$

Joint work with Ana C. Matos and Franck Wielonsky.

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Amilcar Branquinho, University of Coimbra (Portugal)

Title: "Algebraic theory of multiple orthogonal polynomials."

Abstract:
In this talk we present the general theory of multiple orthogonal polynomials. Our departure point is the three term recurrence relation, with matrix coefficients, satisfied by a sequence of vectors polynomials. Connection with the operator theory and constructive theory of approximation are presented.

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Claude Brezinski, University of Lille (France)

Title: "Numerical analysis methods in web search."

Abstract:
An important problem in Web search is to determine the importance of each page. After introducing the main characteristics of this problem, we will see that, from the mathematical point of view, it could be solved by computing the left principal eigenvector (the PageRank vector) of a matrix related to the structure of the Web by using the power method. We will give

expressions of the PageRank vector, and study the mathematical properties of the power method. Various Pad style approximations of the PageRank vector will be given. Since the convergence of the power method is slow, it has to be accelerated. This problem will be discussed. Recently, several acceleration methods were proposed. We will give a theoretical justification to these methods. In particular, we will generalize the recently proposed Quadratic Extrapolation, and we interpret it on the basis of the method of moments of Vorobyev, and as a Krylov subspace method. Acceleration results are given for the various epsilon-algorithms, and for the E-algorithm. Other algorithms for this problem are also discussed.

This is a joint work with M. Redivo-Zaglia.

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Victor Buslaev, Steklov Institut, Moscow (Russia)

Title:" On Fabry's theorem for m-th row of the table of generalized Padé approximants"

Abstract:

Theorem 1 (Fabry) *Let*

$$f(z) = \sum_{n=0}^{\infty} f_n z^n$$

be a power series such that there exists the limit $\lim_{n\to\infty} f_n/f_{n+1} = \lambda \neq 0$. *Then the series converges in the circle* $\{|z| < |\lambda|\}$ *and* λ *is a singular point of the function* $f(z)$ *defined by this series.*

The first part of the Fabry's theorem is an immediate consequence of the Couchy-Adamar's formula for radius of convergence of the power series, but the second part of the Fabry's theorem is a very deep and hardly proving assertion.

Let's remark that f_n/f_{n+1} is a pole of the Pade approximant $[n/1]_f$. S.Suetin extended Fabry's theorem for m-th row of the Pade table.

Theorem 2 (Suetin) *Let*

$$f(z) = \sum_{n=0}^{\infty} f_n z^n$$

be a power series such that the Pade approximants $[n/m]_f$ *have exactly* m *poles* $\lambda_{n,1}, \ldots, \lambda_{n,m}$ *and there exist the limits* $\lim_{n\to\infty} \lambda_{n,j} = \lambda_j \neq 0$, $j = 1, \ldots, m$. *Then the series converges in the circle* $\{|z| < \min_{1\leq j\leq m} |\lambda_j|\}$, *the function* $f(z)$ *defined by this series has meromorphic extension to the circle* $\{|z| < \max_{1\leq j\leq m} |\lambda_j|\}$, $\lambda_1, \ldots, \lambda_m$ *are singular points of the function* $f(z)$ *and there are not other poles in the circle* $\{|z| < \max_{1\leq j\leq m} |\lambda_j|\}$.

It'll be showed in the talk that Suetin's theorem is true for multipointed Padé approximants and Padé approximants with respect to orthogonal series.

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Irina Egorova, University of Kharkov (Ukraine)

Title: "Cauchy problem for Toda lattice with asymptotically periodic initial data"

Abstract:
We study the inverse scattering transform for the Toda lattice for periodic background solution and solve the associated initial value problem in the case when the perturbation of the background has the first moment finite.

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Leonid Golinskii, University of Kharkov (Ukraine)

Title: "Lieb-Thirring and Bargmann-type inequalities for circular arc."

Abstract:
For measures on the unit circle with convergent Verblunsky coefficients we find relation in the form of inequalities between these coefficients and the distances form the mass points of orthogonality measure to the essential support of this measure.

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Sophie Grivaux, University of Lille (France)

Title: "Universal functions for some uncountable families of composition operators on $H^\infty(\mathbb{D})$"

Abstract:
Let ϕ be an automorphism of the unit disk $\mathbb{D}$, and let C_ϕ be the associated composition operator $f \mapsto f \circ \phi$ on $H^\infty(\mathbb{D})$. A function f in the unit ball of $H^\infty(\mathbb{D})$ is a *universal* function for C_ϕ if the orbit $\{C_\phi^n f \; ; n \geq 0\}$ is locally uniformly dense in the unit ball of $H^\infty(\mathbb{D})$. After reviewing some known results regarding the existence of such universal functions, we present some

new facts about the existence of common universal functions (such as Blaschke products, singular inner functions - here universality is considered with respect to the set of zero-free functions in the the unit ball of $H^\infty(\mathbb{D})$, etc...) for some uncountable families of composition operators C_ϕ. This talk is based on joint work with Frédéric Bayart (Bordeaux) and Raymond Mortini (Metz).

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Arno Kuijlaars , Katholieke Universiteit Leuven (Belgium)

Title:" Critical behavior in random matrix models"

Abstract:

I will discuss the behavior of eigenvalues of large Hermitian random matrices in certain critical regimes, related to possible changes in the number of intervals in the limiting spectrum as the size of the matrices tends to infinity. In the critical regimes local eigenvalue correlation functions are described by Painleve transcendents.

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Francisco Marcellan, Universidad Carlos III de Madrid (Spain)

Title:"High Order Linear Difference Equations and nonstandard Orthogonal Polynomials"

Abstract:

It is very well known that orthogonal polynomials associated with a probability measure supported on the real line satisfy a three-term recurrence relation (second order linear difference equation) translating the fact that the multiplication operator by x is a symmetric operator with respect to the standard inner product defined by such a measure. The converse of this result is the so- called Favard's Theorem. Furthermore, a basic set of solutions of such a difference equation is constituted by the above sequence of orthogonal polynomials and the so-called sequence of associated polynomials of the first kind.

Let $\{\mu_k\}_{k=0}^m$ be a vector of positive Borel measures supported on the real line. In the linear space $\boldsymbol{P}$ of polynomials with real coefficients we introduce the following Sobolev inner product

$$\langle p, q\rangle_S := \sum_{k=0}^{m} \int_{\boldsymbol{R}} p^{(k)}(x)\, q^{(k)}(x)\, d\mu_k(x), \tag{1}$$

In general, the sequence of polynomials orthogonal with respect to the above inner product does not satisfy a recurrence relation involving a fixed number of terms, independently of the

degree of the polynomials. In [3] the authors proved that there exists a polynomial multiplication operator H, symmetric with respect to (1), if and only if $\{\mu_k\}_{k=1}^m$ are discrete measures supported in subsets of the real line, associated with the zeros of the polynomial H. As a straightforward consequence, the corresponding sequence of orthogonal polynomials is a solution of a linear difference equation with order depending of the degree of H. The converse result was analyzed in [1] as well as in [2] in the framework of the theory of matrix orthogonal polynomials.

The aim of this contribution is to present some recent results on the basic set of solutions of the above high order linear difference equations (see [4]) as well as to formulate some open problems.

References

[1] A. J. Duran, A generalization of Favards theorem for polynomials satisfying a recurrence relation, *J. Approx. Theory,* **74** (1993), 83–109.

[2] A.J. Duran and W. Van Assche, Orthogonal matrix polynomials and higher order recurrence relation, *Linear Algebra and its Appl.,* **219** (1995), 261–280.

[3] W. D. Evans, L. L. Littlejohn, F. Marcellan, C. Markett, and A. Ronveaux, On recurrence relations for Sobolev orthogonal polynomials *SIAM J. Math. Anal.,* **26** (1995), 446–467.

[4] F. Marcellan and S. M. Zagorodnyuk, On the basic set of solutions of a high order linear difference equation, *J. Diff. Eq. and Appl.,* **12** (2006), 213-228.

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M. Shcherbina, University of Kharkov (Ukraine)

Title: "Universality for orthogonal ensembles of random matrices."

Abstract:

The asymptotic coincidence of reprodusing kernels for unitary and orthogonal ensembles of random matrices will be discussed.

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Christophe Smet, University of Leuven (Belgium)

Title:" An upper bound for the measure of irrationality of $\zeta_q(2)$"

Abstract: We show how one can use Hermite-Padé approximation and little q-Jacobi polynomials to construct rational approximants for $\zeta_q(2)$, which is the q-analogue of the well known $\zeta(2)$. Here $q = \frac{1}{p}$ and p is an integer greater than one. These approximants are good enough to show the irrationality of $\zeta_q(2)$, and they allow us to calculate an upper bound for its measure of irrationality: $\mu(\zeta_q(2)) \leq \frac{10\pi 2}{5\pi 2-24} \approx 3.8936$. This is sharper than the upper bound 4.07869374 given by Zudilin [*On the irrationality measure for a q-analogue of* $\zeta(2)$, Mat. Sb. **193**, no. 8 (2002), 49– 77; Sb. Math. **193**, no. 7–8 (2002), 1151–1172].

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Vladimir Sorokin, University of Moscow (Russia)

Title:"On Zudilin-Rivoal theorem"

Abstract:
There obtained new proof and some complements for the Zudilin-Rivoal theorem on the values of Dirichlet β- function

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Herbert Stahl, TFH Berlin (Germany)

Title:"On the asymptotic behavior of orthonormal polynomials"

Abstract:
In the talk we will be concerned with recent work about the asymptotic behavior of the measures

$$p_n(\mu;\cdot)^2 d\mu \quad \text{for} \quad n \to \infty,$$

where μ is a positive measure with compact support $S = \text{supp}(\mu) \subset \mathbb{R}$, and $p_n = p_n(\mu;\cdot)$ the othonormal polynomial of degree n with respect to the measure μ.

Under the assumptions that $S = \text{supp}(\mu)$ consists of finitetly many intervals and that

$$\frac{d\mu(x)}{dx} > 0 \quad \text{for almost all} \quad x \in S, \tag{\#}$$

we have achieved a proof of the weak convergence

$$p_n(\mu;\cdot)^2 d\mu \xrightarrow{*} \omega_S \quad \text{for} \quad n \to \infty \tag{*}$$

with ω_S denoting the equilibrium distributions on S.

A dream result would be that the convergence (*) holds true for any measure μ with compact support $S = \text{supp}\,(\mu) \subset \mathbb{R}$ of positive capacity satisfying the condition

$$\frac{d\mu}{d\omega_S}(x) > 0 \quad \text{for} \;\; \omega_S - \text{almost all} \quad x \in S,$$

which is a direct generalisation of (#). So far, however, this conjecture remains a dream, since key elements of the available proof depend so strongly on geometric considerations that an extension to the more general situation of an arbitrary set S is still not in sight.

Key elements of the proof of (*) will be addressed and connections with related results will be discussed.

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Walter Van Assche, Jonathan Coussement, University of Leuven (Belgium)

Title: "Differential equations for multiple orthogonal polynomials""

Abstract:

We obtain a lowering operator for multiple orthogonal polynomials having orthogonality conditions with respect to $r \in \mathbb{N}$ classical weights. These multiple orthogonal polynomials are generalizations of the classical orthogonal polynomials. Combining the lowering operator with the raising operators, which have been obtained earlier in the literature, we then also obtain a linear differential equation of order $r+1$.

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J. Van Iseghem, University of Lille (France)

Title: "A (forgotten sequence of rational approximants with quick convergence"