Periodic Orbits In Contact and Riemannian Geometry

September, 3rd - September, 7th, 2012 - Palais des Congrès du Touquet, France

Ivan Babenko - Université de Montpellier

Two lectures on "Systolic geometry: basic constructions and some open problems"

After main definitions, we will consider the systolic problem in frames of smooth manifolds, Riemannian polyhedrons and homology classes. Few open problems will be stated and discussed.

Gérard Besson - Université de Grenoble

Entropy and some related questions

Frédéric Bourgeois - Université Libre de Bruxelles

Two lectures on "Contact geometry"

Oliver Fabert - Freiburg Universität

Qualitative Weinstein conjectures and multiply-covered holomorphic curves

From Hofer´s pioneering work on the Weinstein conjecture it is known that holomorphic curves can be used to prove the existence of a closed Reeb orbit, together with some information about its index. In order to be able to distinguish between different types of orbits (elliptic, hyperbolic, etc.), one however needs to retrieve more information from the linearized flow, like the indices of iterates of the given orbit. If the holomorphic curves are regular by automatic transversality, such additional information can be obtained by studying their multiple covers. Using the resulting local version of symplectic field theory, we prove the existence of an elliptic orbit on certain contact hypersurfaces in four-dimensional symplectic blow-ups.

Dmitri Faifman - Tel-Aviv University

The quotient girth of normed spaces, and Schaffer's dual girth conjecture for higher Grassmannians

The study of the Finsler geometry of the surface of a convex body, which it induce on itself as a submanifold of its normed space, was probably initiated by Schaffer. He made a few beautiful conjectures, among them the following: dual convex bodies have equal girth. This, and the more general equality of length spectra, was proved by Alvarez-Paiva in 06'. We propose a different natural way of associating a Finsler structure to convex bodies, which uses quotient spaces rather than subspaces. It turns out that a dual girth theorem holds also for this construction. We will then show it has a natural extension to higher Grassmannians.

Urs Frauenfelder - Seoul National University

Contacting the moon

This is joint work with P.Albers, G.Paternain and O.van Koert. We prove that energy hypersurfaces of the restricted three body problem below and slightly above the first critical value are of restricted contact type. In this talk I explain this construction and its relation to the search of periodic orbits via global surfaces of section.

Daniel Massart - Université de Montpellier

On the intersection form of surfaces.

The algebraic intersection of closed, oriented curves on an oriented surface M induces a symplectic form on the first homology of M. When M is endowed with a Riemannian metric, we ask the following question : how much can two curves of a given length intersect ? This amounts to computing the norm of the intersection form, with respect to a specific norm on the first homology of M, defined by H. Federer who called it stable norm. In this talk we review what is currently known about the norm of the intersection form, and ask some questions.

Marco Mazzucchelli - ENS Lyon

Symplectically degenerate maxima via generating functions

In this talk, we will discuss a new approach to the study of symplectically degenerate maxima by means of generating functions techniques. In particular, we will provide a simple proof of a theorem due to Nancy Hingston, asserting that isolated symplectically degenerate maxima of a Hamiltonian diffeomorphism of a standard symplectic torus are non-isolated points of its average-action spectrum.

Klaus Niederkrüger - Université de Toulouse

Existence of certain submanifolds in higher dimensional contact manifolds and the Weinstein conjecture

This is joint work with P.Ghiggini, P.Massot and C.Wendl. Hofer showed in '93 that the Weinstein conjecture is true for any contact 3-manifold with non-vanishing second homotopy group. In previous work with Ana Rechtman, we showed that Hofer's proof can be modified to higher dimensions for certain submanifolds having a Legendrian foliation and representing a non-trivial homology class. In this talk I show how to strengthen the arguments to show that in the special case of dimension five and the submanifold being a 3-sphere, it suffices for the sphere to be a non-trivial class in homotopy instead of homology.

Emmanuel Opshtein - Université de Strabourg

Lagrangian connectors, symplectic embeddings, and C^0 rigidity

Gromov-Eliashberg C^0 rigidity theorem allows to define the notion of a symplectic homeomorphism. The natural question after this theorem is to understand which properties the symplectic homeomorphisms share with their smooth cousins. In this talk, I will explain some results in this direction, and their connections to the problems of symplectic embeddings.

Yaron Ostrover - Tel-Aviv University

Convex billiards – a symplectic geometry point of view

In this talk we shall discuss how a certain symplectic invariant on the classical phase space can be used to obtain bounds and inequalities on the length of the shortest periodic billiard trajectory in a convex domain. Moreover, we shall explain how the above approach applies both for the classical (Euclidean) case and the more general case of Minkowski billiards. This talk is based on a joint work with Shiri Artstein-Avidan from Tel-Aviv University.

Hugo Parlier - Université de Fribourg

Systolic inequalities and kissing numbers for surfaces

Schmutz Schaller introduced a nice parallel between, on the one hand the study of Hermite constants and kissing numbers for n dimensional lattices, and on the other hand the study of optimal systolic inequalities and upper bounds on the number of systoles that a hyperbolic surface of genus g can have. As in the case of lattices, only a finite number of optimal such inequalities are known. It is natural to ask what the asymptotic behavior of these constants might be. Buser and Sarnak constructed surfaces with near optimal asymptotic systole growth (around 4/3 log(g)) and Schmutz Schaller constructed examples of surfaces with relatively large number of systoles (roughly g^{4/3}). In this talk I'll discuss these inequalities, and some related inequalities on closed Riemannian surfaces and explain why hyperbolic surfaces of genus g cannot have more than roughly g^2 systoles.

Federica Pasquotto - VU Amsterdam University

Periodic orbits on non-compact hypersurfaces in cotangent bundles

In this talk I will present an existence result for periodic orbits on a fixed non-compact energy level of a mechanical Hamiltonian system on the cotangent bundle of a Riemannian manifold. I will focus on the analytical and geometrical problems posed by the lack of compactness and discuss advantages and limitations of our method of proof ("linking"). This is joint work with T. Rot, J.B. van den Berg and R. Vandervorst.

Ana Rechtman - Université de Strasbourg

Existence of periodic orbits of geodesible vector fields on 3-manifolds

A non-singular vector field on a closed manifold M is geodesible if there is a Riemannian metric making its orbits geodesics. We will study the existence of periodic orbits for such vector fields on closed 3-manifolds. On 3-manifolds, K. Kuperberg constructed examples of vector fields without periodic orbits. On the other hand, C. H. Taubes proved that the Reeb vector field of a contact form has periodic orbits. Reeb vector fields are geodesible, and also suspensions are geodesible. If we assume that the ambient manifold M is either diffeomorphic to the three-sphere or has a non trivial second homotopy group, we will prove the existence of a periodic orbit for volume preserving geodesible vector fields.

Stéphane Sabourau - Université de Créteil

Critical isosystolic surfaces

We will show that the two piecewise flat surfaces with conical singularities conjectured by Calabi as extremal surfaces for the isosystolic problem in genus 3 are critical with respect to some metric variations.

Alfonso Sorrentino - Cambridge University

Two lectures on "The principle of least action in Hamiltonian dynamics"

In these two lectures I shall present John Mather's variational approach to the study of convex and superlinear Hamiltonian systems, what is generally called Aubry-Mather theory. Starting from the crucial observation that invariant Lagrangian graphs can be characterised in terms of their "action-minimizing properties", we shall investigate how analogue features can be traced in a more general setting, namely the so-called Tonelli Hamiltonian systems. This different point of view will bring to light a plethora of compact invariant subsets of the system that, under many points of view, could be considered as generalisation of invariant Lagrangian graphs, despite not being in general either submanifolds or regular. We shall discuss their structure and their symplectic properties, as well as their relation to the dynamics of the system. Moreover, time permitting, I shall point out some connections of this theory to other topics, such as classical mechanics, Hamilton-Jacobi equation (weak KAM theory), symplectic geometry, Hofer's geometry etc...

Kai Zehmisch - Köln Universität

A new capacity

In my talk I will report on a joint project with Hansjörg Geiges on the construction of a symplectic capacity, which can be used to estimate the smallest period of closed Reeb orbits from above as well as to prove the (strong) Weinstein conjecture for a rich class of contact manifolds.