It is claimed in the main theorem of [2] (Theorem 1.1) that
\[ \text{gr } V_1(\mathfrak{sl}_n) \cong C[J_\infty \overline{\mathfrak{s}_1}] \quad \text{and} \quad \text{gr } V_m(\mathfrak{sl}_{2m}) \cong C[J_\infty \overline{\mathfrak{s}_0}] \]
as Poisson vertex algebras, but this is not true in general (the rest of the theorem is correct).

Indeed, using Macaulay2 one can compute the Hilbert series of \( J_1 \overline{\mathfrak{s}_1} \), and we obtain for \( n = 4 \) that
\[ H(J_1 \overline{\mathfrak{s}_1}; q) = 1 + 15q + 115q^2 + 620q^3 + 2785q^4 + o(q^4). \]
On the other hand, from the results of [1] it is possible to obtain an explicit character formula of \( \text{gr } V_1(\mathfrak{sl}_4) \) as in [3], and we obtain for \( n = 4 \) that
\[ \chi_{V_1(\mathfrak{sl}_4)}(q) = 1 + 15q + 115q^2 + 620q^3 + 2765q^4 + o(q^4). \]

Hence \( J_\infty \overline{\mathfrak{s}_1} \) and \( \text{SS}(V_1(\mathfrak{sl}_4)) \) are not isomorphic as schemes.

The error comes from the fact that the isomorphisms of Theorem 4.1 of [2] hold only as reduced schemes. However, in Corollary 4.2 of [2] the arc spaces \( J_\infty \overline{\mathfrak{o}} \) and \( J_\infty \overline{\mathfrak{s}} \) needs not to be reduced. In addition, the hypothesis that the singular support \( \text{SS}(V) \) is \( J_\infty \mathbb{C}^* \)-invariant is necessary in Theorem 4.1 although we show below (see Lemma 3) that this holds for the vertex algebras \( V_1(\mathfrak{sl}_n) \) considered in [2, Th. 1.1].

To summarize, Theorem 1.1 of [2] has to be replaced by:

**Theorem 1** (replacement of [2, Th. 1.1]).

1. For \( n \geq 4 \),
\[ \tilde{X}_{V_1(\mathfrak{sl}_n)} \cong \overline{\mathfrak{s}_1} \]
as schemes, where \( \mathfrak{l}_1 \) is the standard Levi subalgebra of \( \mathfrak{sl}_n \) generated by all simple roots except \( \alpha_1 \). Moreover \( V_1(\mathfrak{sl}_n) \) is a quantization of the infinite jet scheme \( J_\infty \overline{\mathfrak{s}_1} \) of \( \overline{\mathfrak{s}_1} \) in the sense that
\[ \text{SS}(V_1(\mathfrak{sl}_n)) \cong J_\infty \overline{\mathfrak{s}_1} \]
as topological spaces, that is, \( \text{SS}(V_1(\mathfrak{sl}_n))_{\text{red}} \cong (J_\infty \overline{\mathfrak{s}_1})_{\text{red}} \).

2. For \( m \geq 2 \),
\[ \tilde{X}_{V_m(\mathfrak{sl}_{2m})} \cong \overline{\mathfrak{s}_0} \]
as schemes, where \( \mathfrak{l}_0 \) is the standard Levi subalgebra of \( \mathfrak{sl}_{2m} \) generated by all simple roots except \( \alpha_m \).

**Theorem 4.1** of [2] has to be replaced by:

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Theorem 2 (replacement of [2, Th. 4.1]). Let $V$ be a quotient vertex algebra $V^k(\mathfrak{g})$. Suppose that $X_V = \overline{G.C^*x}$ for some $x \in \mathfrak{g}$ and that $J_\infty C^*.x$ is contained in the reduced singular support $SS(V)_{\text{red}}$, where the action of $J_\infty C^*$ on $J_\infty \mathfrak{g}$ is induced by the $C^*$-action on $\mathfrak{g}$. Then
\[
SS(V) = J_\infty X_V = J_\infty \overline{G.C^*x} = J_\infty \overline{G.C^*x},
\]
as topological spaces. In particular, $SS(V)_{\text{red}} = (J_\infty X_V)_{\text{red}}$.

The proof is unchanged (except that in the last sentence, “$J_\infty C^*$-invariant” has to be replaced by “contains $J_\infty C^*.x$”, provided that we work on $C$-points. The equality (1) in Corollary 4.2 of [2] still holds but as topological spaces. The second one has to be removed. Moreover, the lemma below (Lemma 3) has to added in order to apply Theorem 2 to the vertex algebra $V_{-1}(\mathfrak{sl}_n)$ we consider in Theorem 1.

Finally, the second sentence in the proof of Theorem 1.1 (1), page 183, has to be replaced by “The second statement follows from Theorem 4.1 (now Theorem 2) and Lemma 3”, the second sentence in the proof of Theorem 1.1 (2), page 195, has to be removed.

Lemma 3. Assume that $V = V_{-1}(\mathfrak{sl}_n(C))$. Then the hypothesis of Theorem 2 are satisfied.

Proof. Let $x \in \mathfrak{g}^* \cong \mathfrak{g}$ be such that $X_V = \bar{SS}_{\min} = \overline{G.C^*x}$.

We have a natural map $\varphi_x : C^* \to X_V$, $t \mapsto t.x$. It gives a morphism from $R_V$ to $O(C^*) = \mathbb{C}[t, t^{-1}]$ which induces a morphism from $J_\infty R_V$ to $O(J_\infty C^*) = \mathbb{C}[t_0, t_0^{-1}, t_1, t_2, \ldots]$. Note that $J_\infty C^*$ is nothing but $(\pi_{\infty,0}^C)^{-1}(C^*)$, where $\pi_{\infty,0}^C$ is the canonical projection from $J_\infty C$ to $C$.

On the other hand, we have a surjective Poisson vertex algebra morphism $\rho : J_\infty R_V \to \text{gr} V$.

The question is to know whether the morphism $J_\infty \varphi_x^* : J_\infty R_V \to O(J_\infty C^*)$ factorizes through $\rho$. Indeed, if so, then it implies that $J_\infty C^*.x$ is contained in $SS(V)_{\text{red}}$. We will prove that it is true for some $x \in \mathfrak{g}$ such that $X_V = \overline{G.C^*x}$.

Let $\hat{\mu}$ be the morphism $\hat{\mu} : \text{gr} V \longrightarrow \mathbb{C}[J_\infty T^*C^n]$ induced from the embedding from $V = V_{-1}(\mathfrak{sl}_n)$ to $\mathcal{D}_{\mathfrak{ch}}^C$ (see [2, Th. 7.13]). Denote by $\mu$ the restriction to $R_V \subset \text{gr} V$ of $\hat{\mu}$. Note that $\mu$ is the morphism from $R_V = \mathbb{C}[\mathfrak{g}]/I$ to $\mathbb{C}[T^*C^n]$ such that the image of $e_{i,j} + I \in R_V$ is $-\beta_j \gamma_i$ and the image of $h_i + I \in R_V$ is $-\beta_i \gamma_i + \beta_{i+1} \gamma_{i+1}$, with $\beta_1, \ldots, \beta_n, \gamma_1, \ldots, \gamma_n$ the natural coordinates on $T^*C^n \cong \mathbb{C}^{2n}$. Let $W \subset O(T^*C^n)$ be the image of $\mu$. Note that we have $J_\infty W \subset O(J_\infty T^*C^n)$. It suffices to show that for some $x$ such that $X_V = \overline{G.C^*x}$, there is a well-defined morphism $\nu_x$ from $W$ to $O(C^*)$ such that $\nu_x \circ \mu = \varphi_x^*$, that is, such that the following diagram commutes:

$$
\begin{array}{ccc}
O(C^*) & \xrightarrow{\nu_x} & W \\
\xrightarrow{\varphi_x^*} & & \xrightarrow{\mu} \\
R_V & & \\
\end{array}
$$
Indeed, assume the statement proven, and pick \( x \) satisfying the above conditions. We observe that \( ^\cdot \circ = J^1 \) (its comes from the identification of \( J^\infty g^* \) with \( S(t^{-1}g[t^{-1}]) \)). Hence, we get the commutative diagram:

\[
\begin{array}{c}
J^\infty RV \\
\rho \downarrow \quad \downarrow J^\infty \mu \\
\text{gr}V \\
\text{J}^\infty \mu \\
\end{array}
\]

Therefore, for such an \( x \), \( J^\infty \phi^*_x \) factorizes through \( \text{gr}V \): we have \( \rho \circ \mu \circ J^\infty \nu_x = J^\infty \phi^*_x \).

To complete the proof, it thus remains to find such an \( x \). It is enough to show that for some \( x \), \( \text{ker} \phi^*_x \).

Recall now that \( \text{Specm}(C^*[g]/\ker \mu) \subset \text{Specm}(C^*[g]/I) = \mathcal{S}_{\text{min}} \). Since \( Z_\mu \) is \( G \)-invariant and \( C^* \)-invariant, it is enough to prove that there is \( x \in G \cdot C^* \cdot 1 \) such that \( \ker \mu \subset \mathfrak{m}_x \) for all \( t \in C^* \). Assume the contrary. We expect a contradiction. We have \( Z_\mu \cap G \cdot C^* \cdot 1 = \emptyset \) by our assumption, and so \( Z_\mu \subset \mathcal{U}_{\text{min}} \). So the defining ideal of \( \mathcal{U}_{\text{min}} \) would be contained in \( \ker \mu \). But this is not true, since the Casimir element,

\[
\Omega = \sum_{1 \leq i \neq j \leq n} e_{i,j} e_{j,i} + \sum_{i=1}^{n} h_i \epsilon_i,
\]

is not in \( \ker \mu \). Indeed, the coefficient of \( \beta_2 \gamma_1 \beta_1 \gamma_2 \) in \( \mu(\Omega) \in C[\beta_i, \gamma_i \mid i = 1, \ldots, n] \) is

\[
2 - 2h_1(\epsilon_1) + h_2(\epsilon_1) + h_1(\epsilon_2)
\]

\[
= \begin{cases} 
2 - \frac{2(n - 1)}{n} + \frac{n - 2}{n} + \frac{n - 2}{n} = \frac{2(n - 1)}{n} & \text{if } n > 4, \\
\frac{3}{2} & \text{if } n = 4.
\end{cases}
\]

This proves the expected statement, and so completes the proof. \( \square \)

REFERENCES


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