

1ère Journée lilloise de proba-stat

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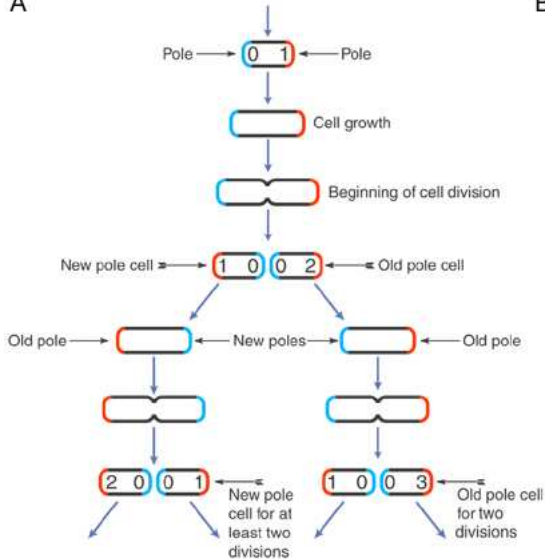
6 janvier 2017

Processus bifurquants appliqués à la biologie

Vincent Bansaye¹, Jean-François Delmas²,
Benoîte de Saporta³, Anne Gégout-Petit⁴,
Laurence Marsalle, Viet Chi Tran⁵

Division d' *E. coli*

A



B

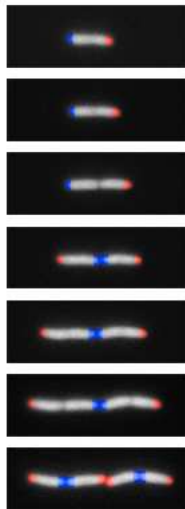
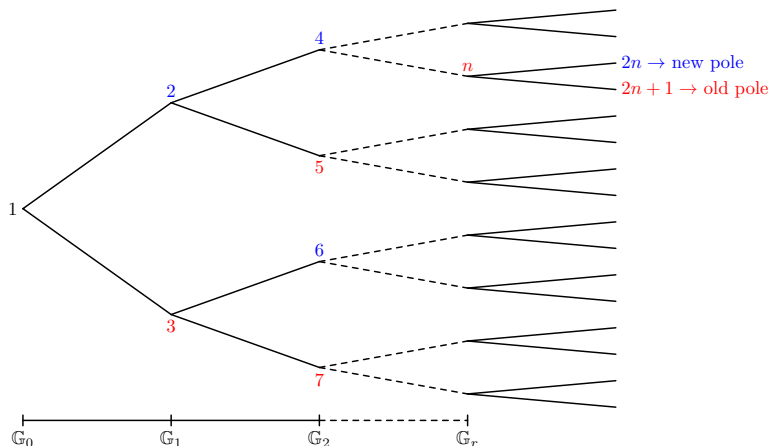


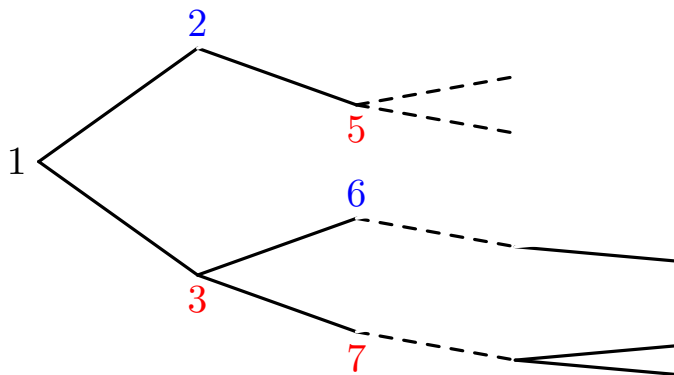
Figure: D'après Stewart *et al.*, 2005

Modélisation de la généalogie



Génération r : $G_r = \{2^r, \dots, 2^{r+1} - 1\}$

Généalogie incomplète : arbres de Galton-Watson



Un modèle statistique basé sur l'arbre incomplet

On note $\tilde{\mathbb{T}}$ le sous-arbre **aléatoire** des cellules dont les 2 filles sont vivantes. Pour $i \in \tilde{\mathbb{T}}$ on pose

$$\begin{cases} X_{2i} &= \alpha_0 X_i + \beta_0 + \varepsilon_{2i}, \\ X_{2i+1} &= \alpha_1 X_i + \beta_1 + \varepsilon_{2i+1}, \end{cases}$$

- $\alpha_0, \alpha_1 \in (-1, 1)$, $\beta_0, \beta_1 \in \mathbb{R}$,
- $((\varepsilon_{2i}, \varepsilon_{2i+1}))$ i.i.d., centrées, gaussiennes, de même variance $\sigma^2 > 0$ et de coefficient de corrélation linéaire $\rho \in (-1, 1)$,
- Pour répondre à la question du vieillissement, on testera $(\alpha_0, \beta_0) = (\alpha_1, \beta_1)$ contre $(\alpha_0, \beta_0) \neq (\alpha_1, \beta_1)$.

Les estimateurs

Estimateurs du maximum de vraisemblance basés sur les données fournies par $r + 1$ générations : pour $\delta \in \{0, 1\}$

$$\hat{\alpha}_\delta^r = \frac{\text{Cov}^r(X_i, X_{2i+\delta})}{\text{Var}^r(X_i)} \quad \text{et} \quad \hat{\beta}_\delta^r = \text{Moy}^r(X_{2i+\delta}) - \hat{\alpha}_\delta^r \text{Moy}^r(X_i),$$

où

$$\text{Moy}^r(X_i) = \frac{1}{|\tilde{\mathbb{T}}_r|} \sum_{i \in \tilde{\mathbb{T}}_r} X_i, \quad \text{Var}(X_i)^r = \frac{1}{|\tilde{\mathbb{T}}_r|} \sum_{i \in \tilde{\mathbb{T}}_r} X_i^2 - (\text{Moy}^r(X_i))^2,$$

$$\text{Cov}^r(X_i, X_{2i+\delta}) = \frac{1}{|\tilde{\mathbb{T}}_r|} \sum_{i \in \tilde{\mathbb{T}}_r} X_i X_{2i+\delta} - \text{Moy}^r(X_i) \text{Moy}^r(X_{2i+\delta}).$$

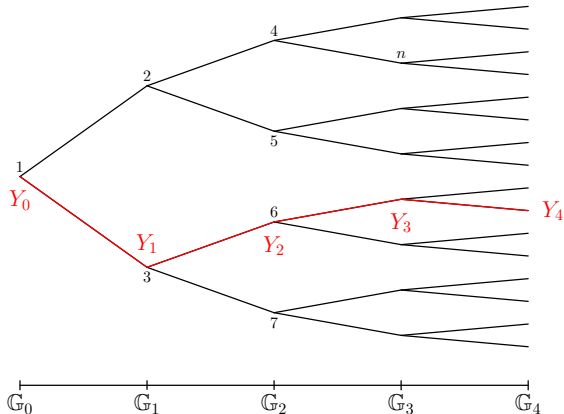
Loi des grands nombres

$$\mathbf{1}_{\{|\mathbf{G}_r^*| > 0\}} \frac{1}{|\mathbf{T}_r^*|} \sum_{i \in \mathbf{T}_r^*} f(X_i, X_{2i}, X_{2i+1}) \xrightarrow[r \rightarrow \infty]{\text{Pr}} \mathbf{1}_{\{W > 0\}} \langle \mu, P^* f \rangle.$$

$$P^* f(x) := \mathbb{E}[f(X_i, X_{2i}, X_{2i+1}) | X_i = x].$$

Loi des grands nombres

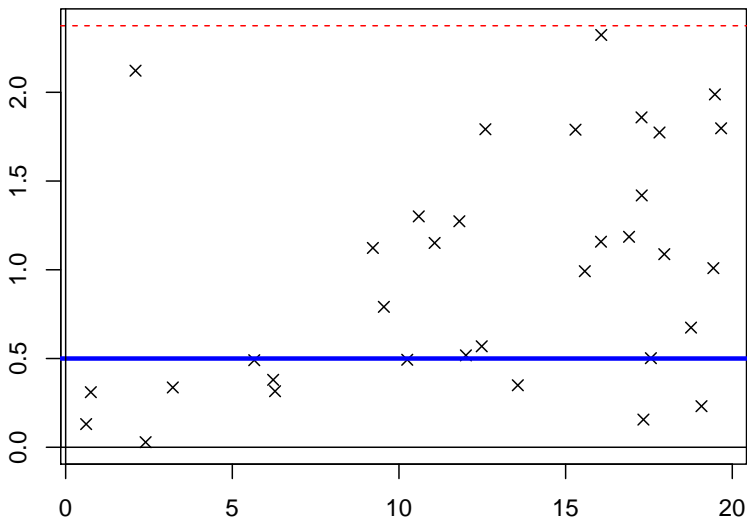
$$\mathbf{1}_{\{|G_r^*|>0\}} \frac{1}{|T_r^*|} \sum_{i \in T_r^*} f(X_i, X_{2i}, X_{2i+1}) \xrightarrow[r \rightarrow \infty]{\text{Pr}} \mathbf{1}_{\{W>0\}} \langle \mu, P^* f \rangle.$$



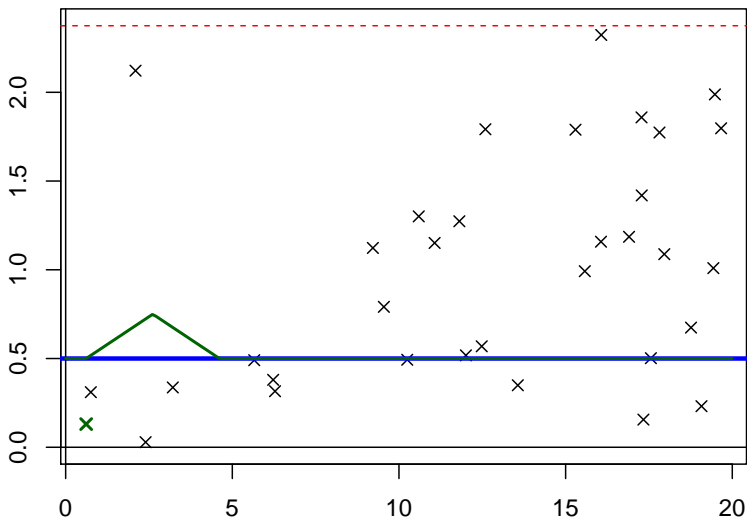
Renouvellement et processus de Hawkes

Manon Costa¹, Carl Graham²,
Laurence Marsalle, Viet Chi Tran³
Patricia Reynaud-Bouret⁴

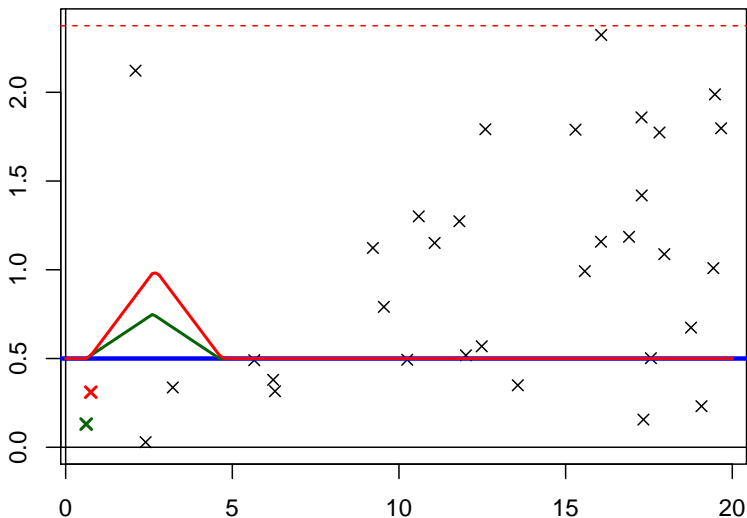
Construction d'un processus de Hawkes sur [0 , 20]



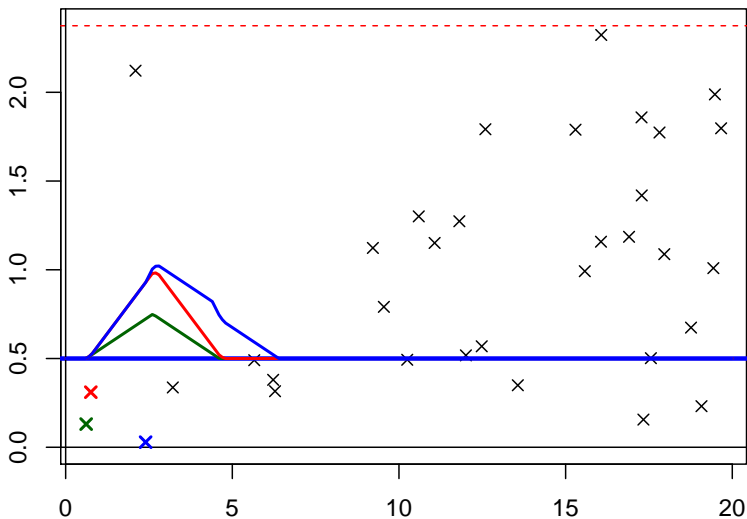
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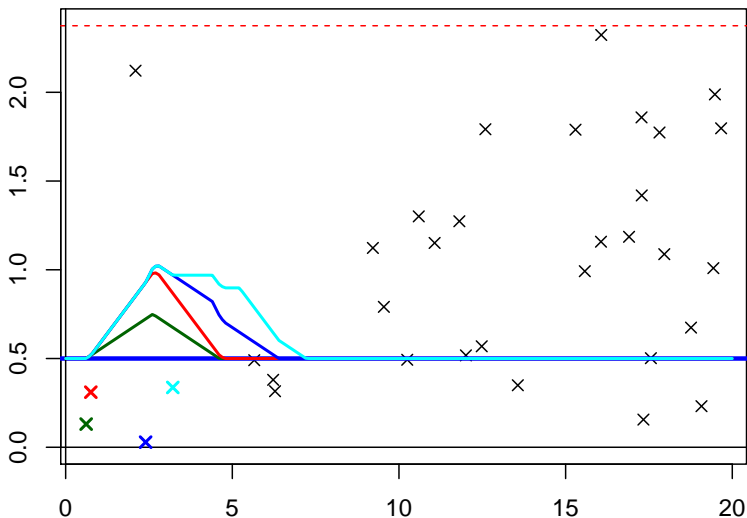
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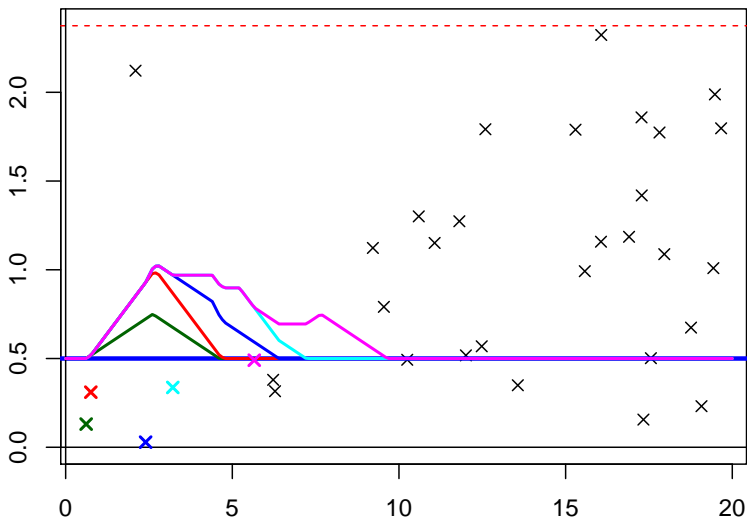
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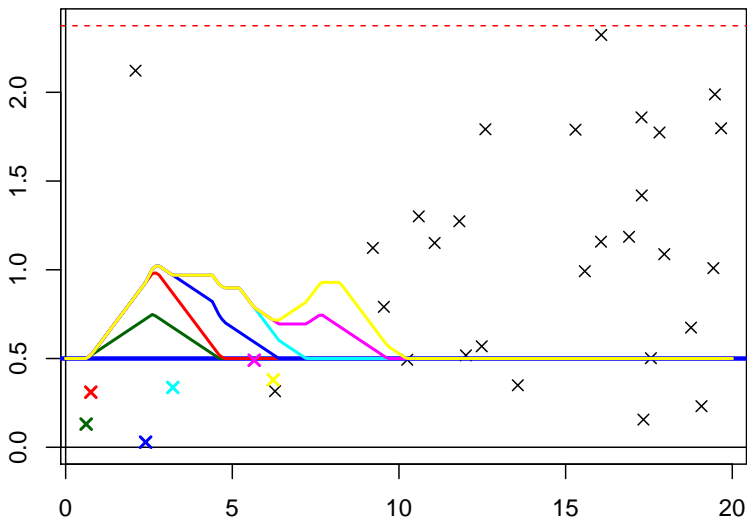
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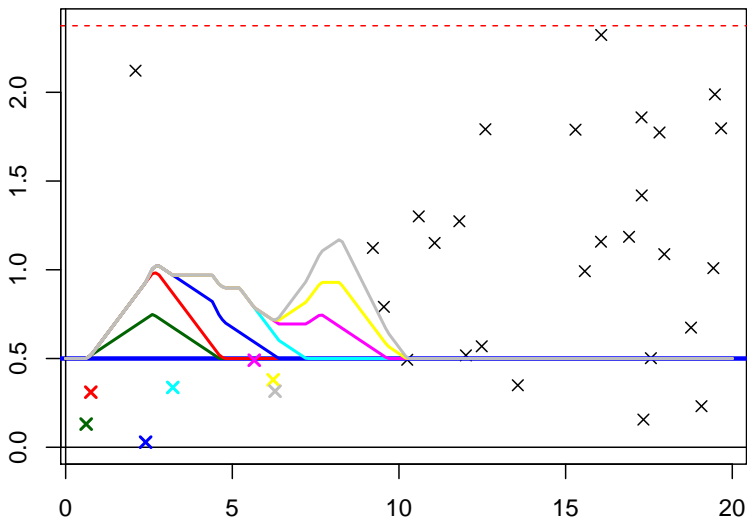
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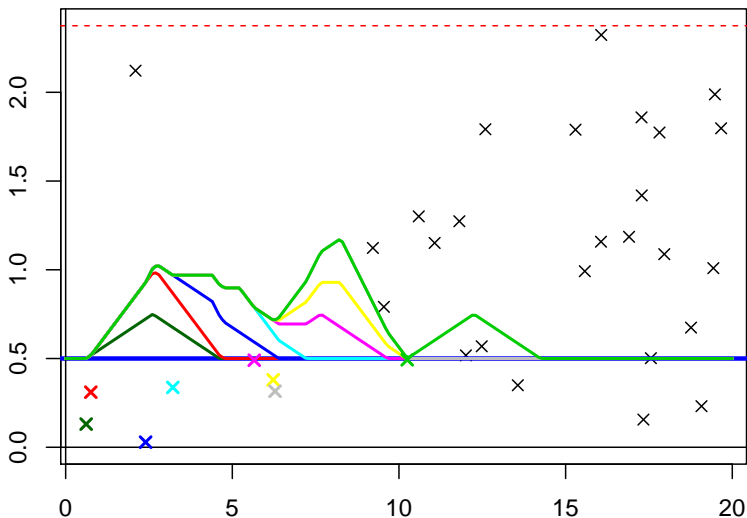
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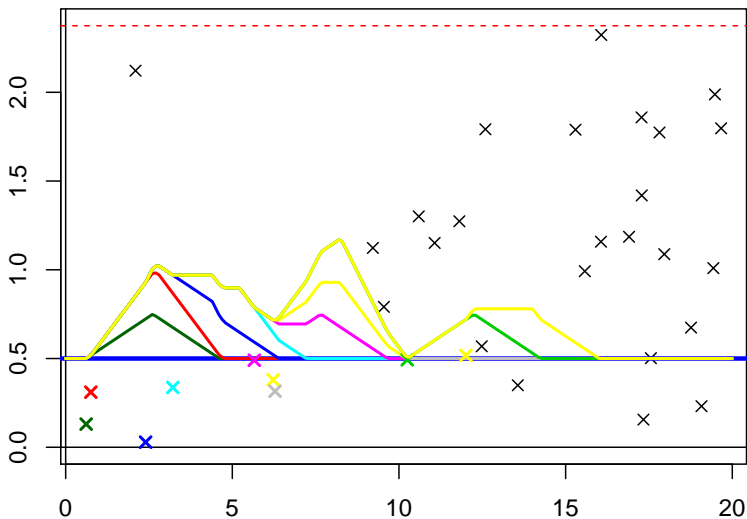
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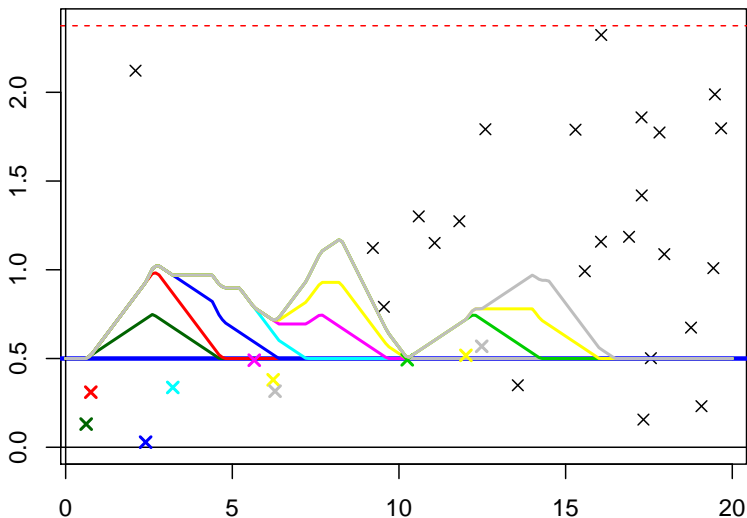
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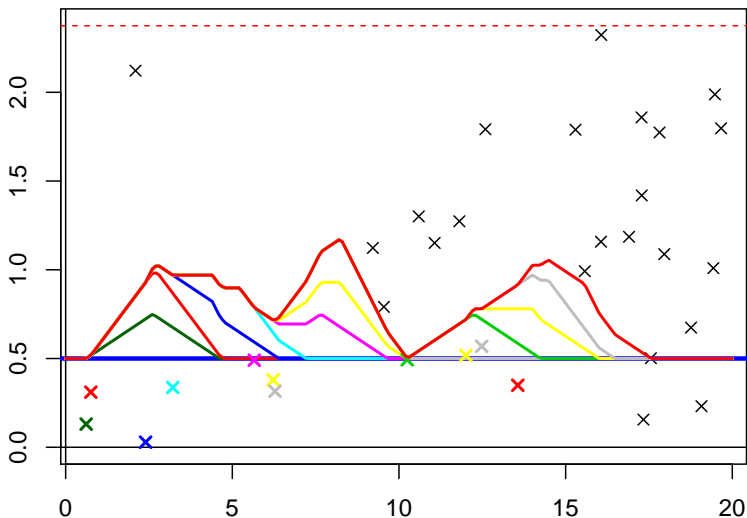
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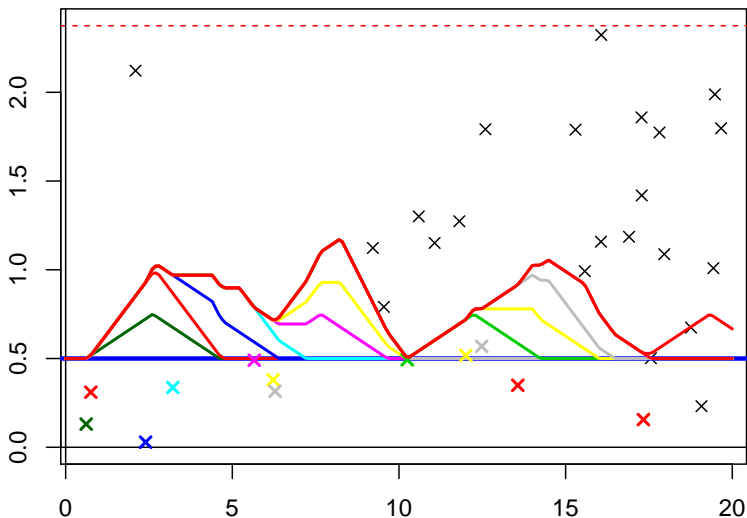
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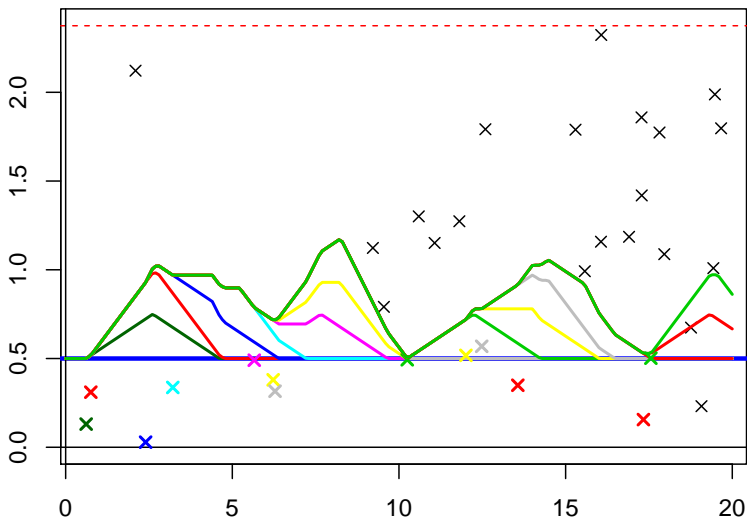
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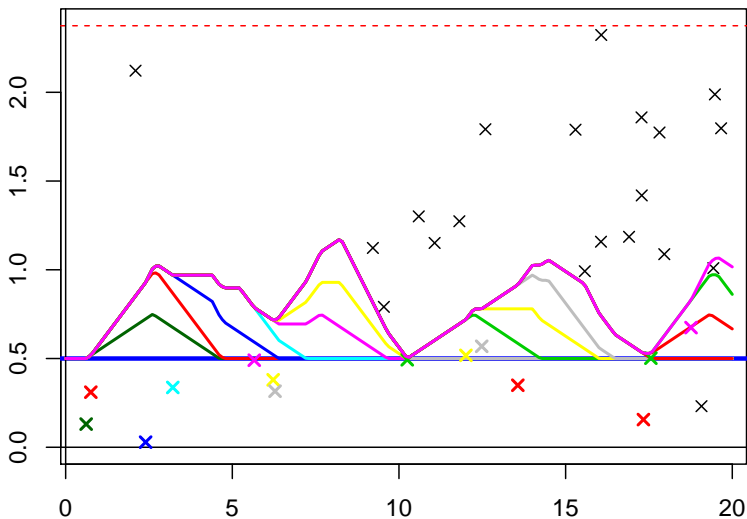
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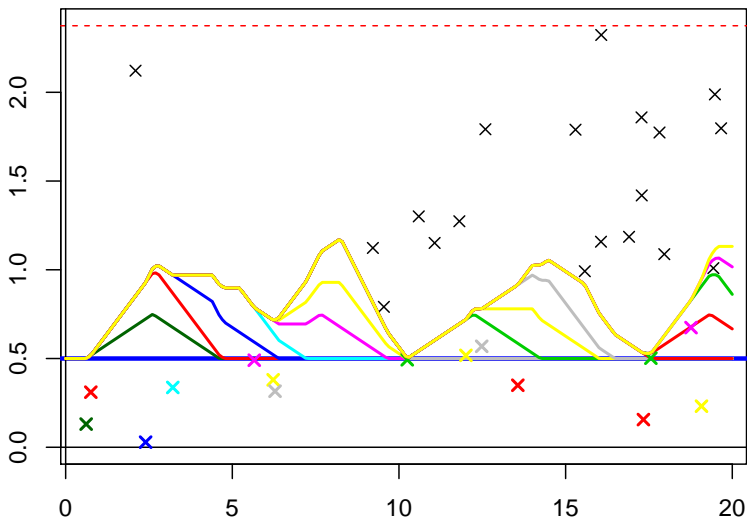
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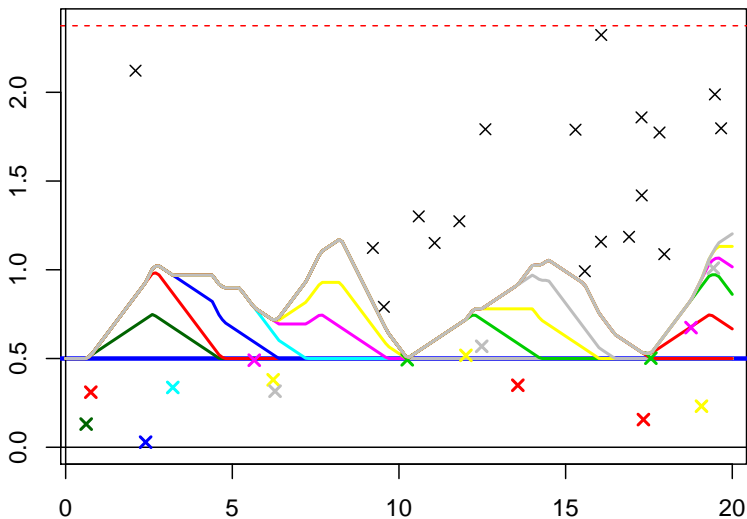
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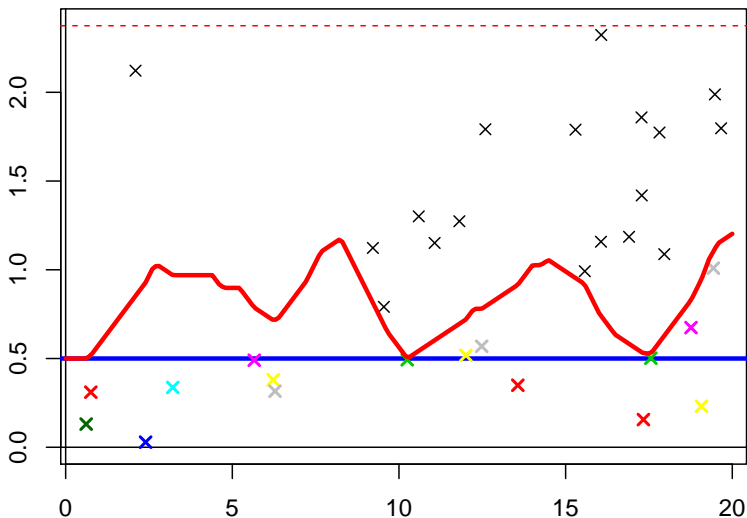
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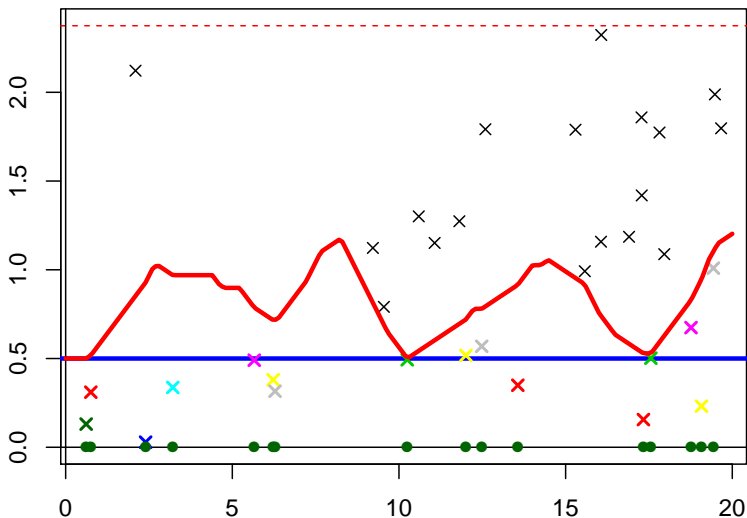
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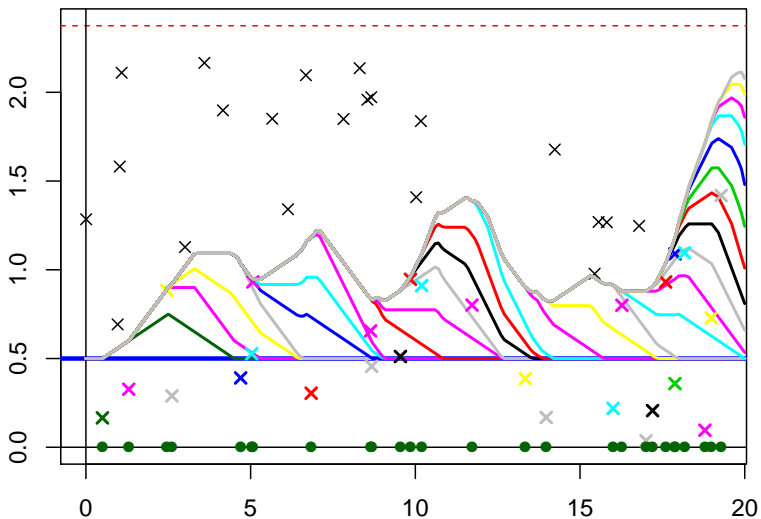
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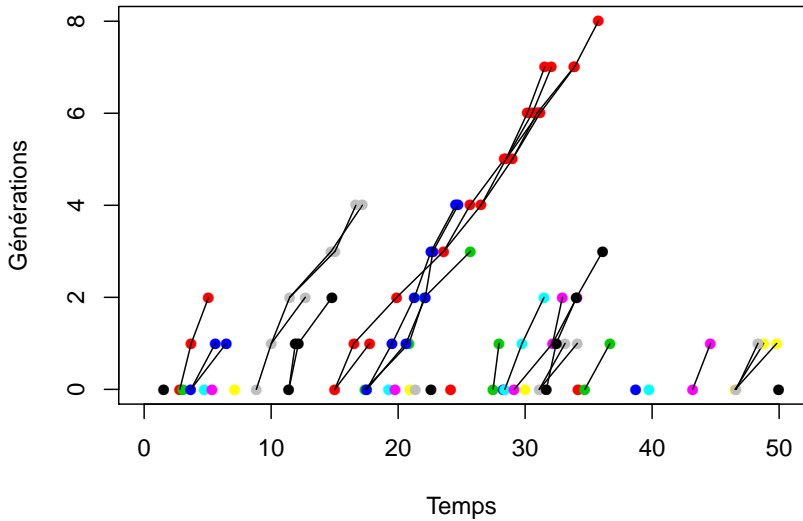


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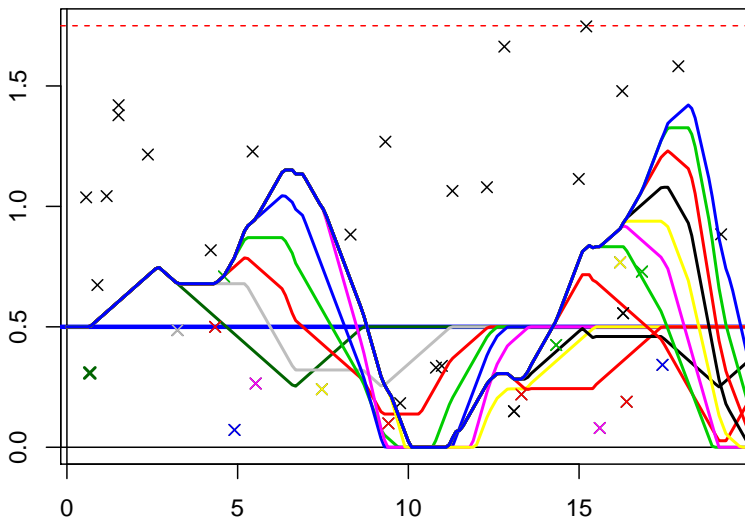


Existence d'une généalogie

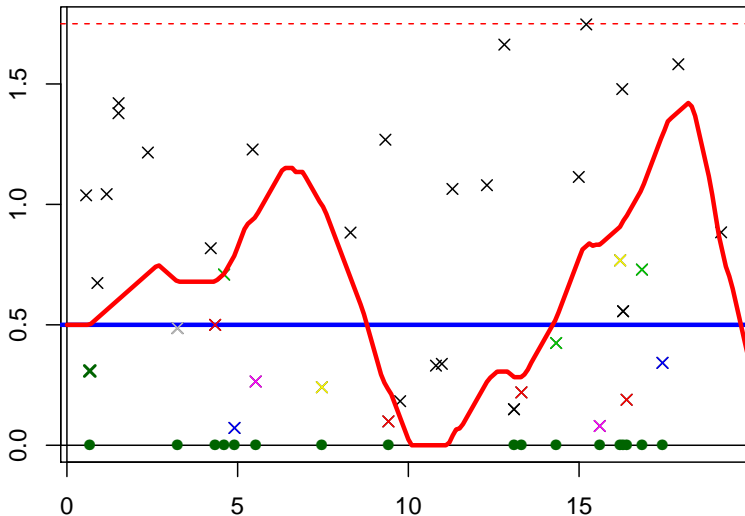




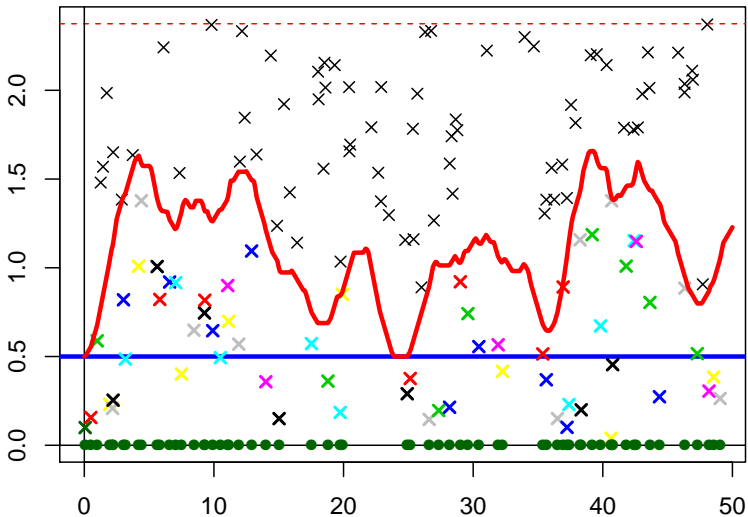
Processus de Hawkes avec h de signe quelconque



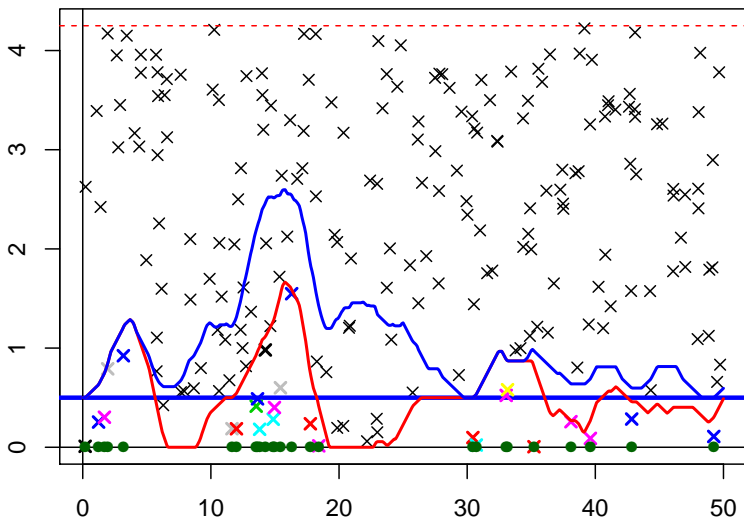
Processus de Hawkes avec h de signe quelconque



Temps de renouvellement



Couplage $H(\lambda, h)$ et $H(\lambda, h+)$





P. Reynaud-Bouret et E. Roy

Some non asymptotic tail estimates for Hawkes processes. *Bull. Belg. Math. Soc. - Simon Stevin*, 13(5):883-896, 2007.



A.G. Hawkes et D. Oakes

A cluster process representation of a self-exciting process. *Journal of Applied Probability*, 11:493-503, 1974.



L. Tackács

On a probability problem arising in the theory of counters. *Proc. Cambridge Philos. Soc.*, 52:488-498, 1956.