# SPECIALIZATION AND LOCALIZATION IN INVERSE GALOIS THEORY

PIERRE DÈBES

*Oberwolfach*, Monday, April 16, 2018

The following pages are the slides of my talk. They display a number of IGT properties for a finite group $G$ over a given number field $k$. The abbreviations used for these properties refer to the glossary given in §2 and §4 from

*Specialization and Localization in Inverse Galois Theory*, Oberwolfach Reports, (2018). `http://math.univ-lille1.fr/~pde/pub.html`— item 57

The implication arrows show the hierarchy between the properties. The groups appearing above a given box satisfy the corresponding property, those appearing below do not, both over $k = \mathbb{Q}$. The symbol ... (resp. $\cdot$) means that the list is open (resp. that it is closed), possibly as a question if used with a question mark. The main recent results are those of these assertions about groups satisfying or not satisfying a property which come with a reference. References are given in the end.

$\underline{G \text{ finite group}}$
$\underline{k \text{ a number field}}$

$\underline{G \text{ finite group}}$
$\underline{k \text{ a number field}}$

RIGP

$\xRightarrow[\text{HIT}]{}$

IGP

$\underline{G \text{ finite group}}$
$\underline{k \text{ a number field}}$

$S_n\,, A_n$
$abelian\,, D_{2n}$
$many\ simple\ldots$
RIGP

$\Longrightarrow$
HIT

IGP

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$many\ simple...$
RIGP

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$solvable$
IGP

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. . . ?

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IGP
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$G$ has a generic extension $F/k(T)$

$\Downarrow$

$G$ has a parametric extension $F/k(T)$

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*$S_n$, $A_n$*
*abelian, $D_{2n}$*
*many simple...*
RIGP
. . . ?

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HIT

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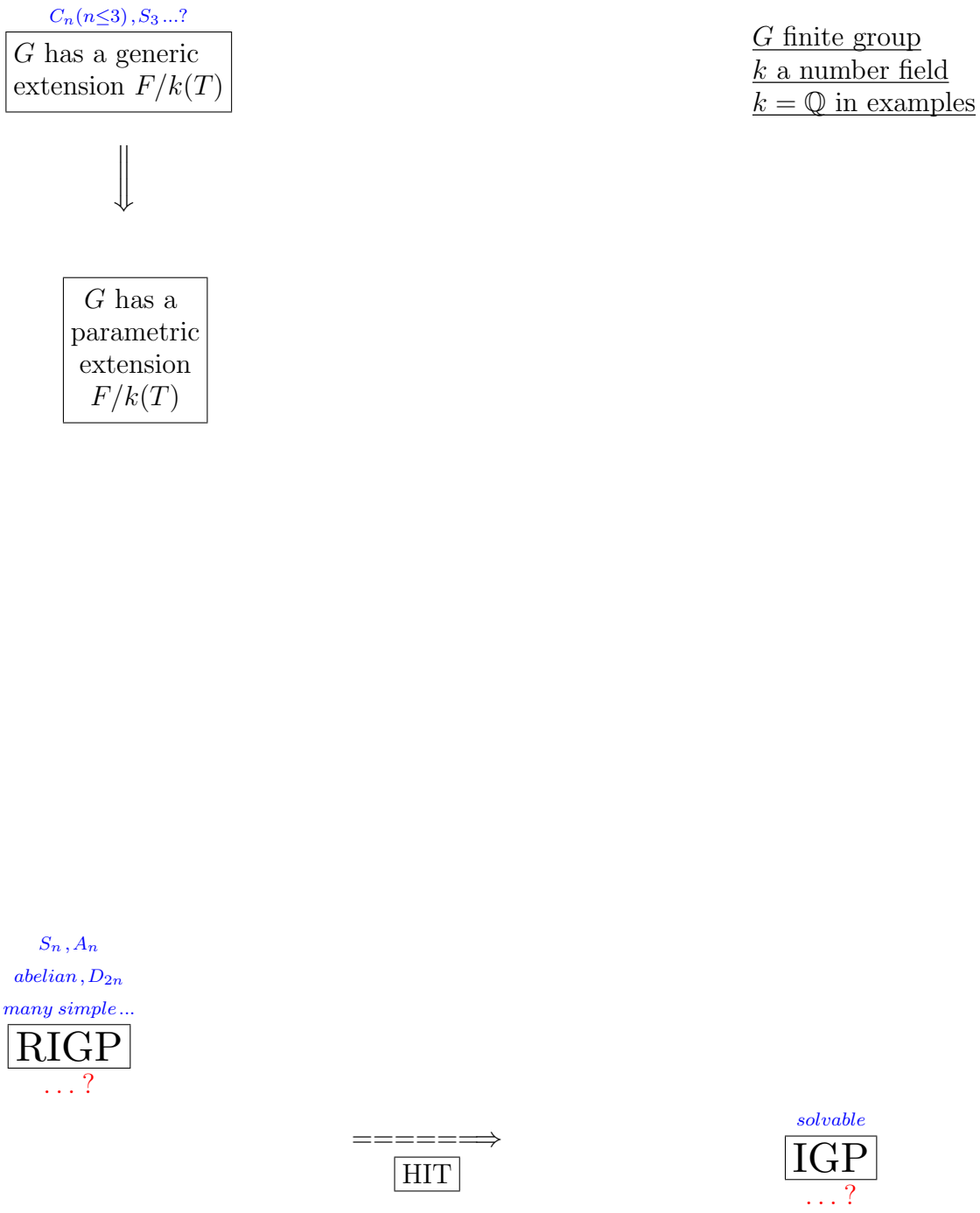
IGP
. . . ?


$C_n (n \leq 3), S_3 ...?$
$G$ has a generic extension $F/k(T)$
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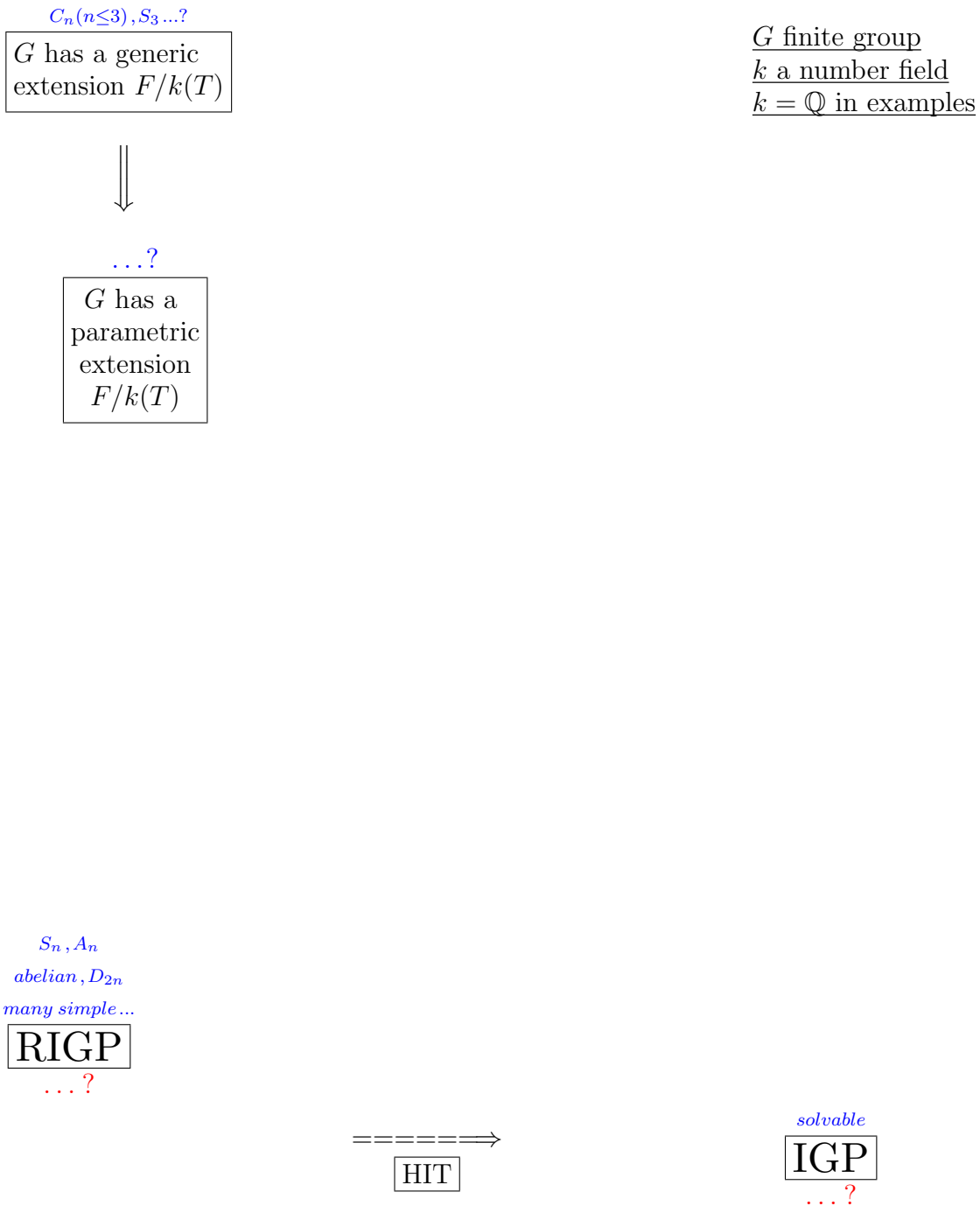
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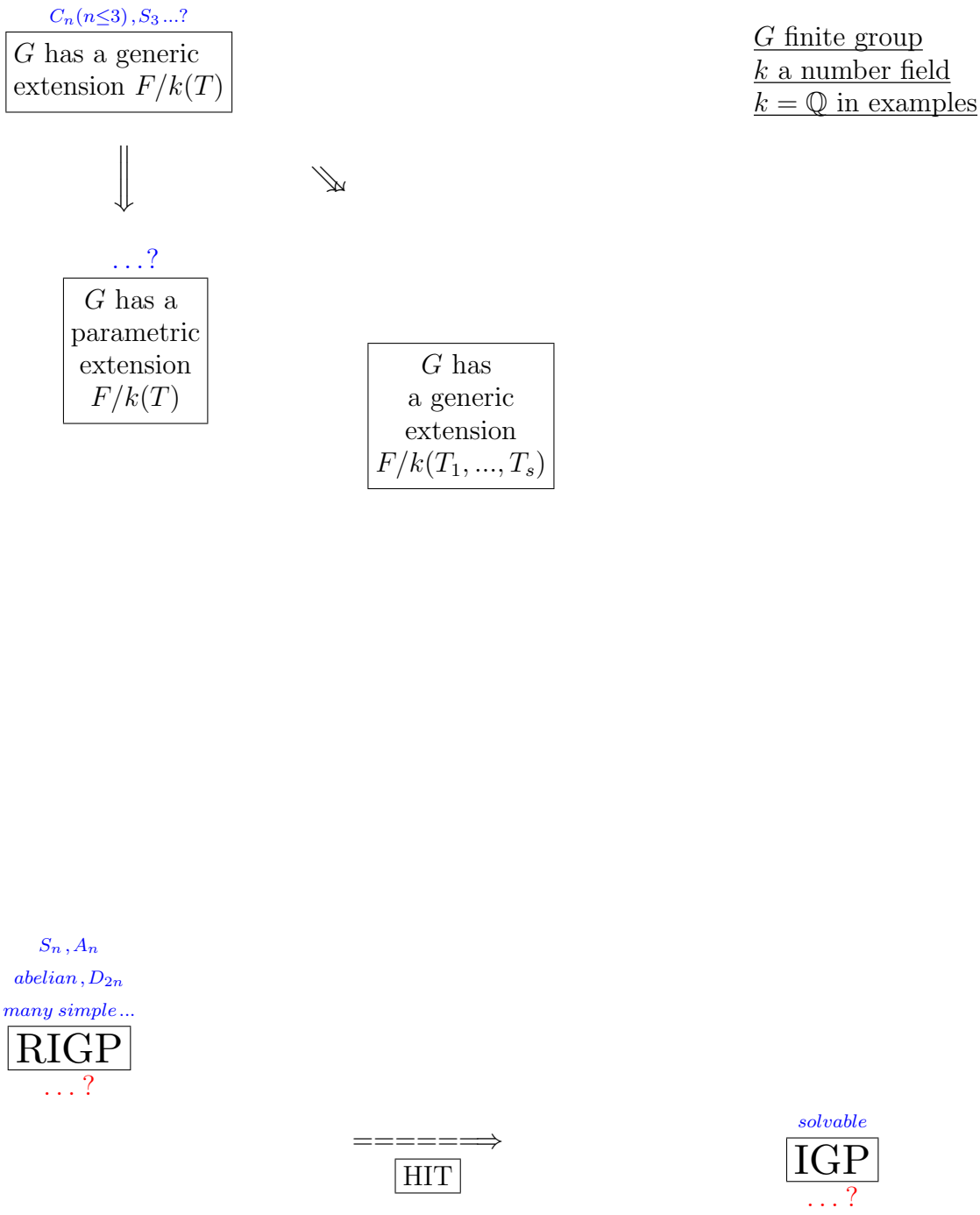
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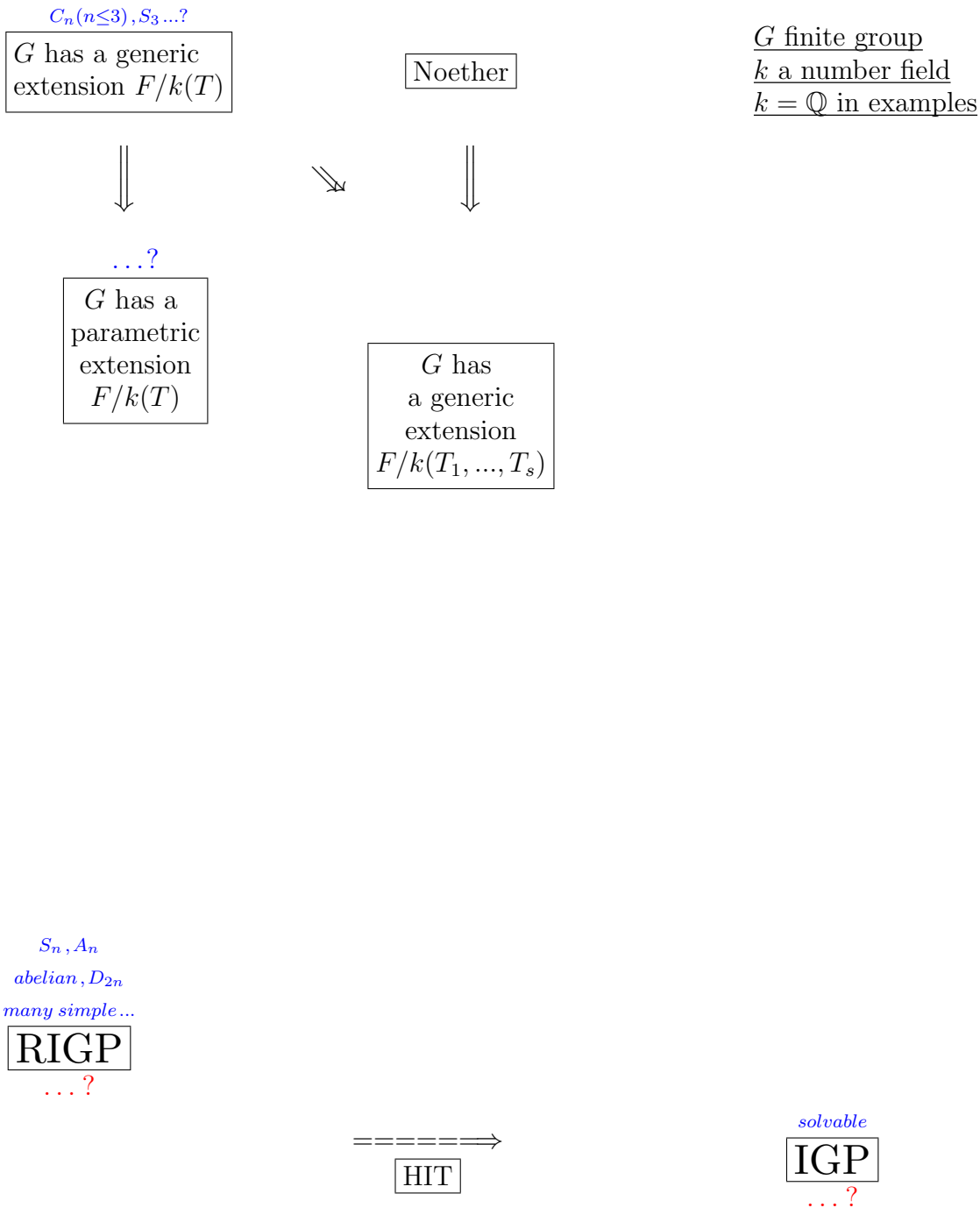
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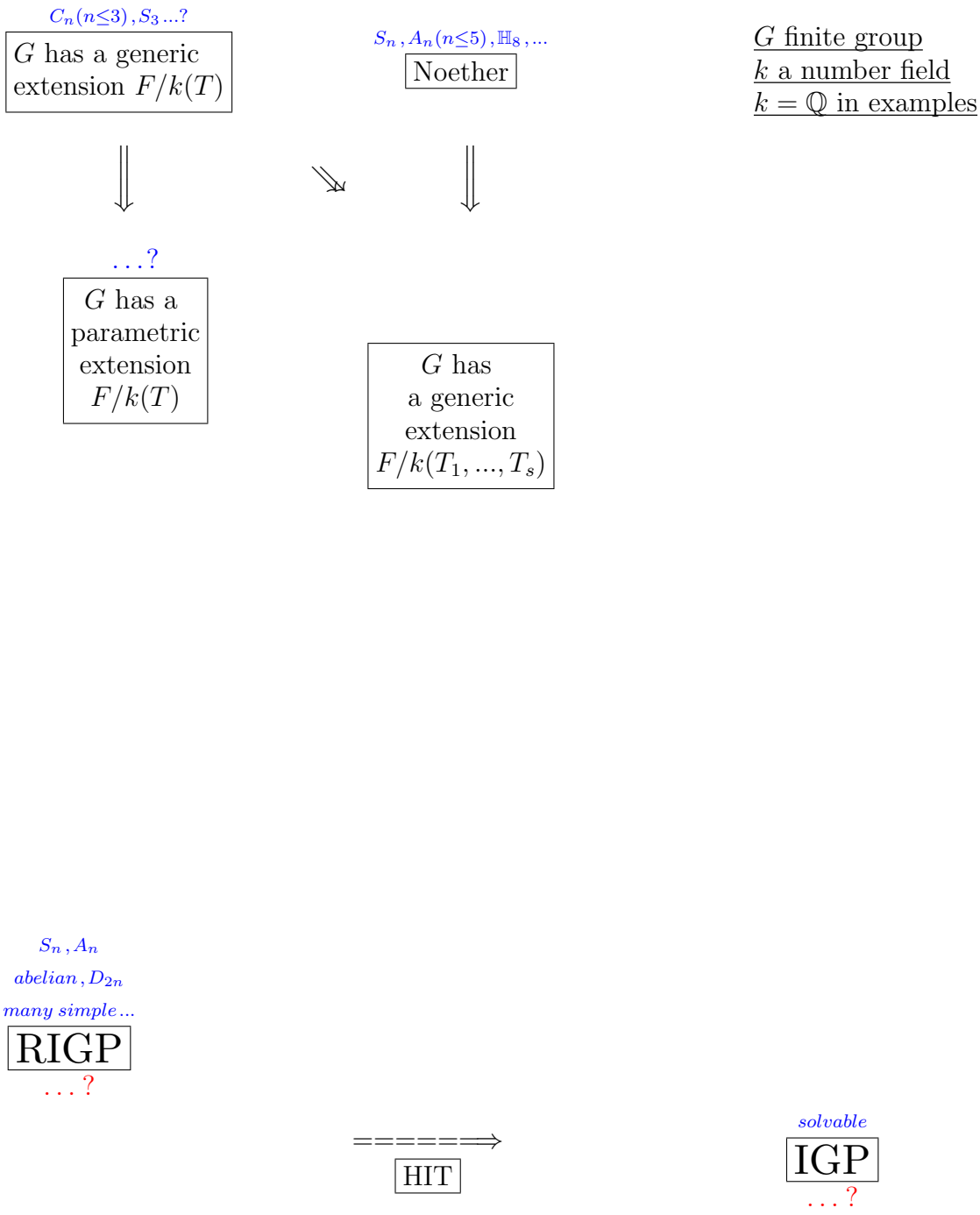
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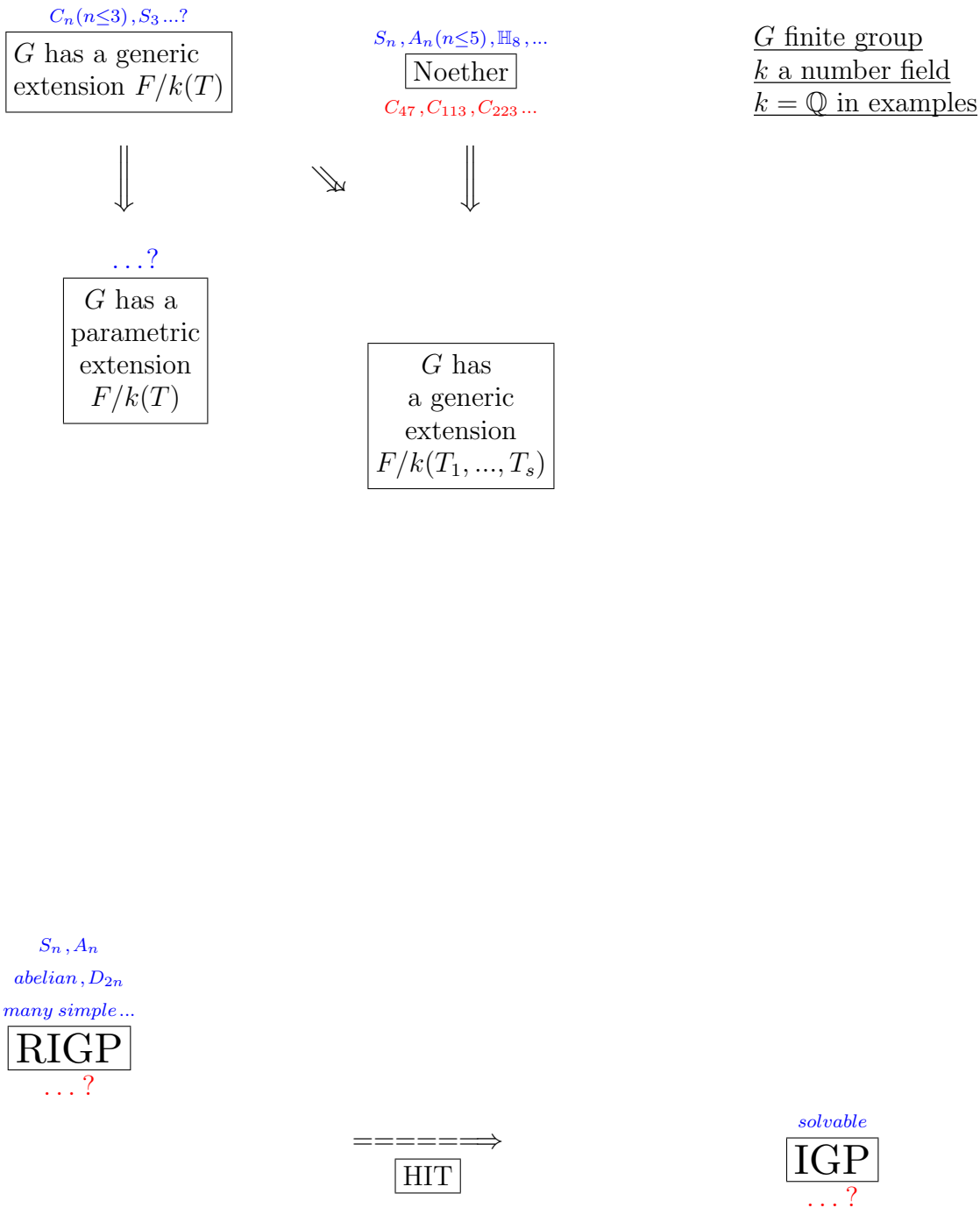
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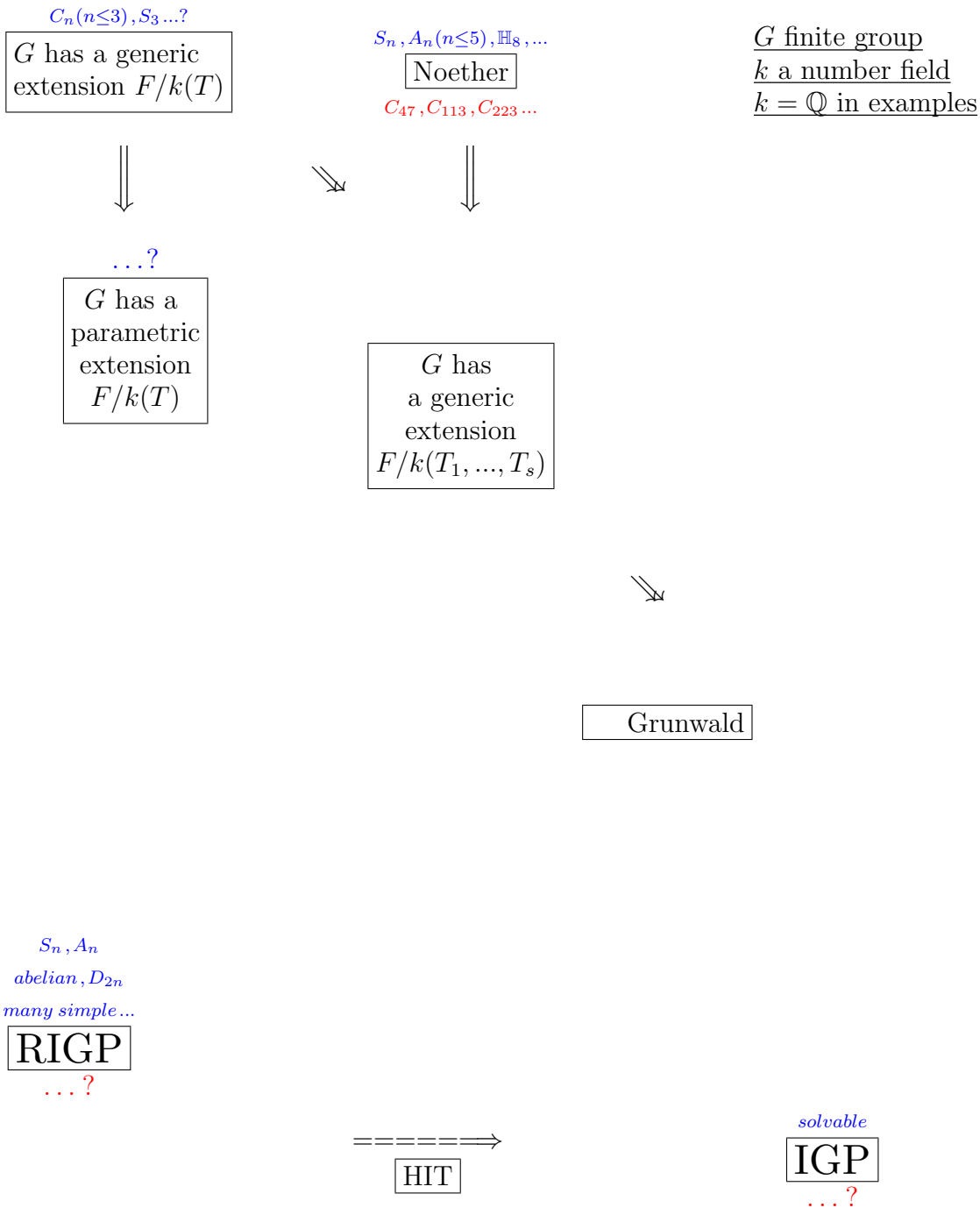
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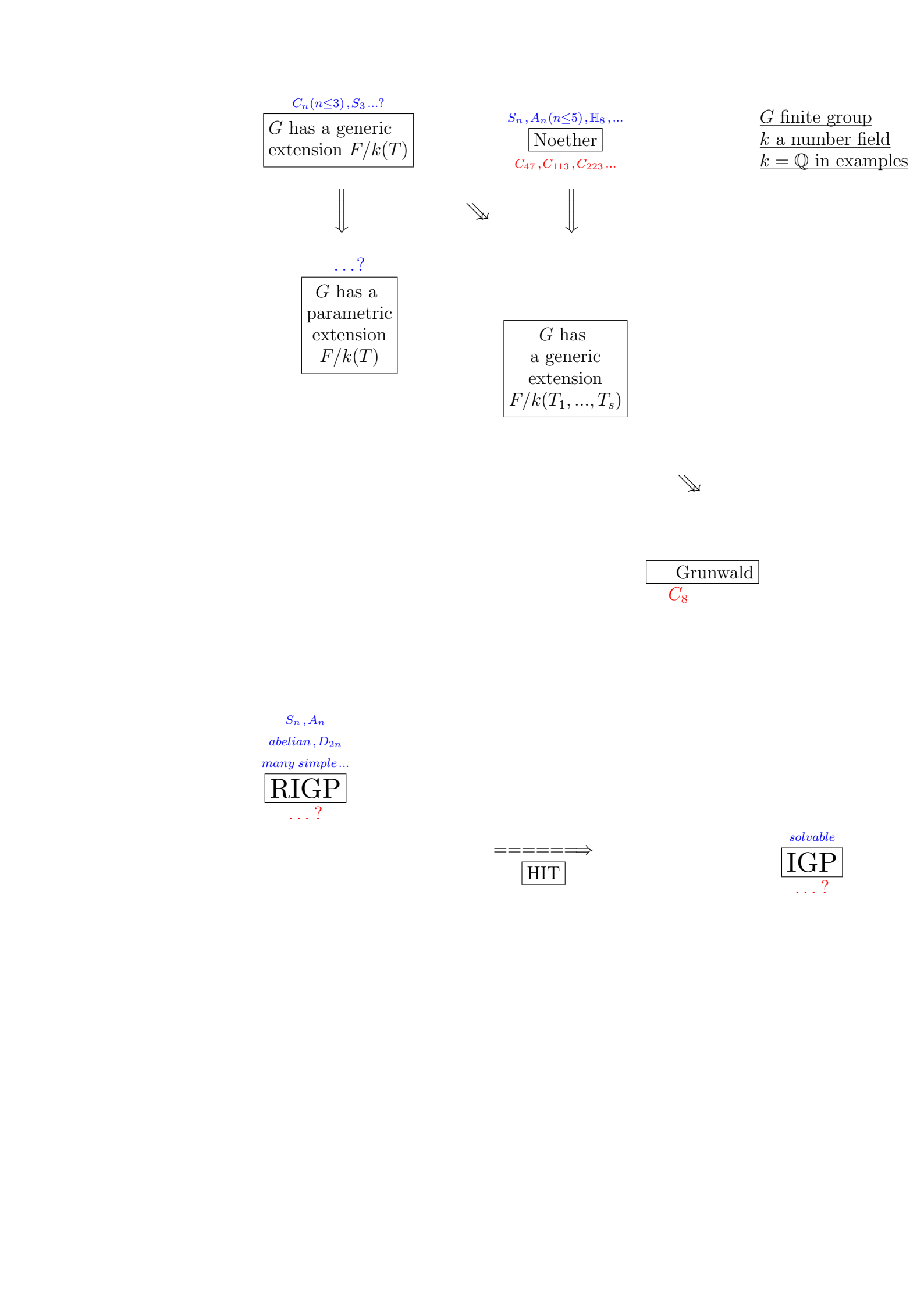
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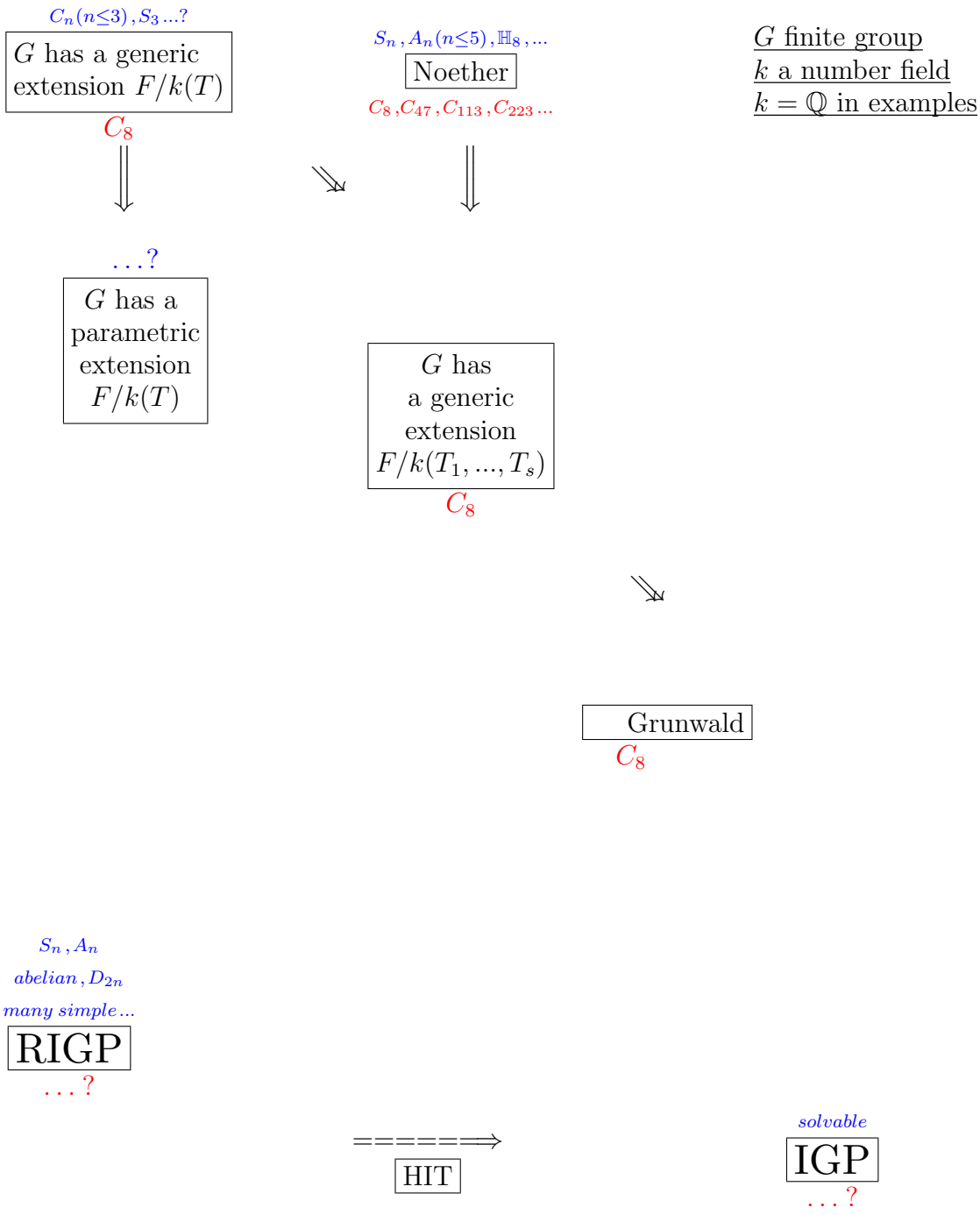
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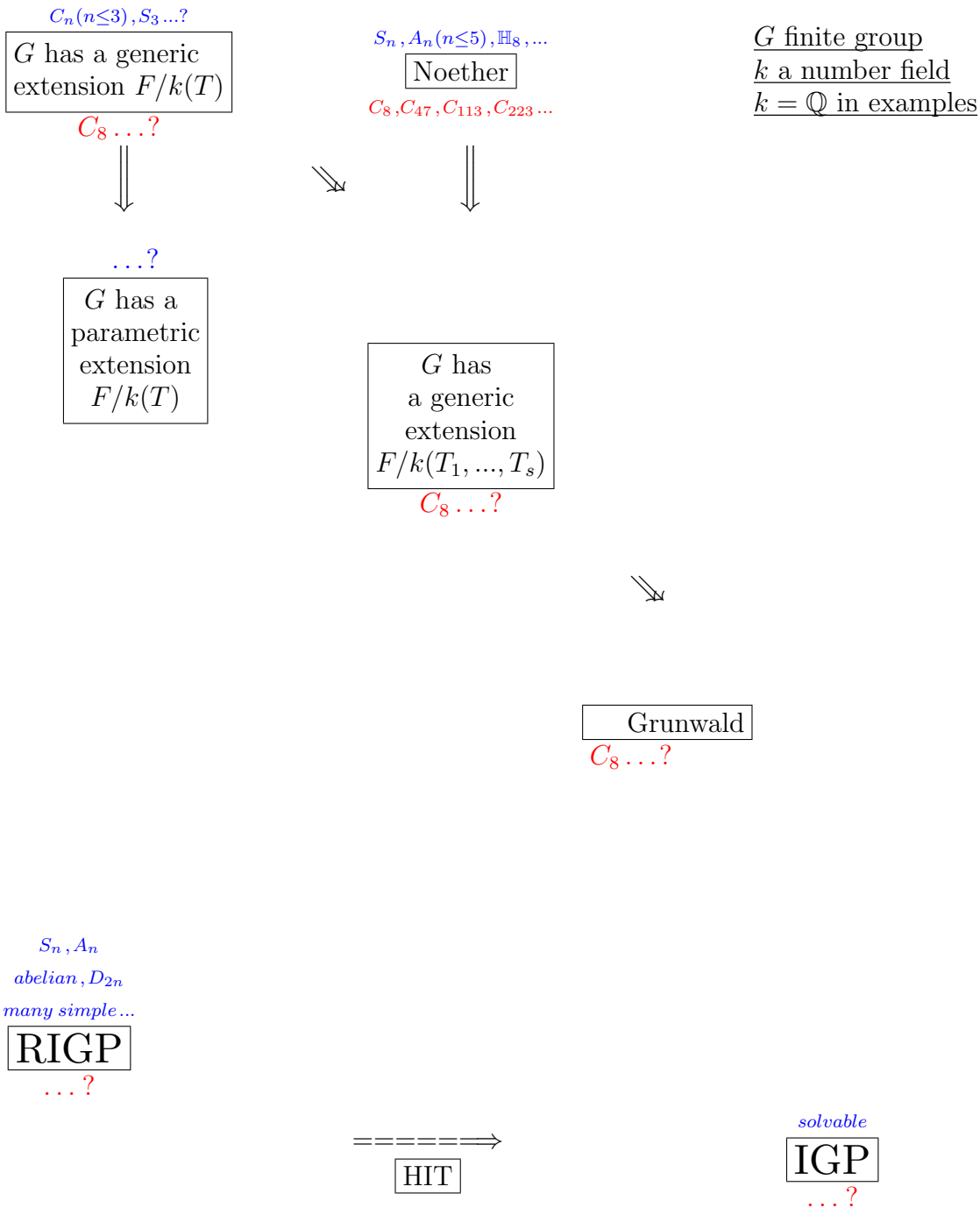
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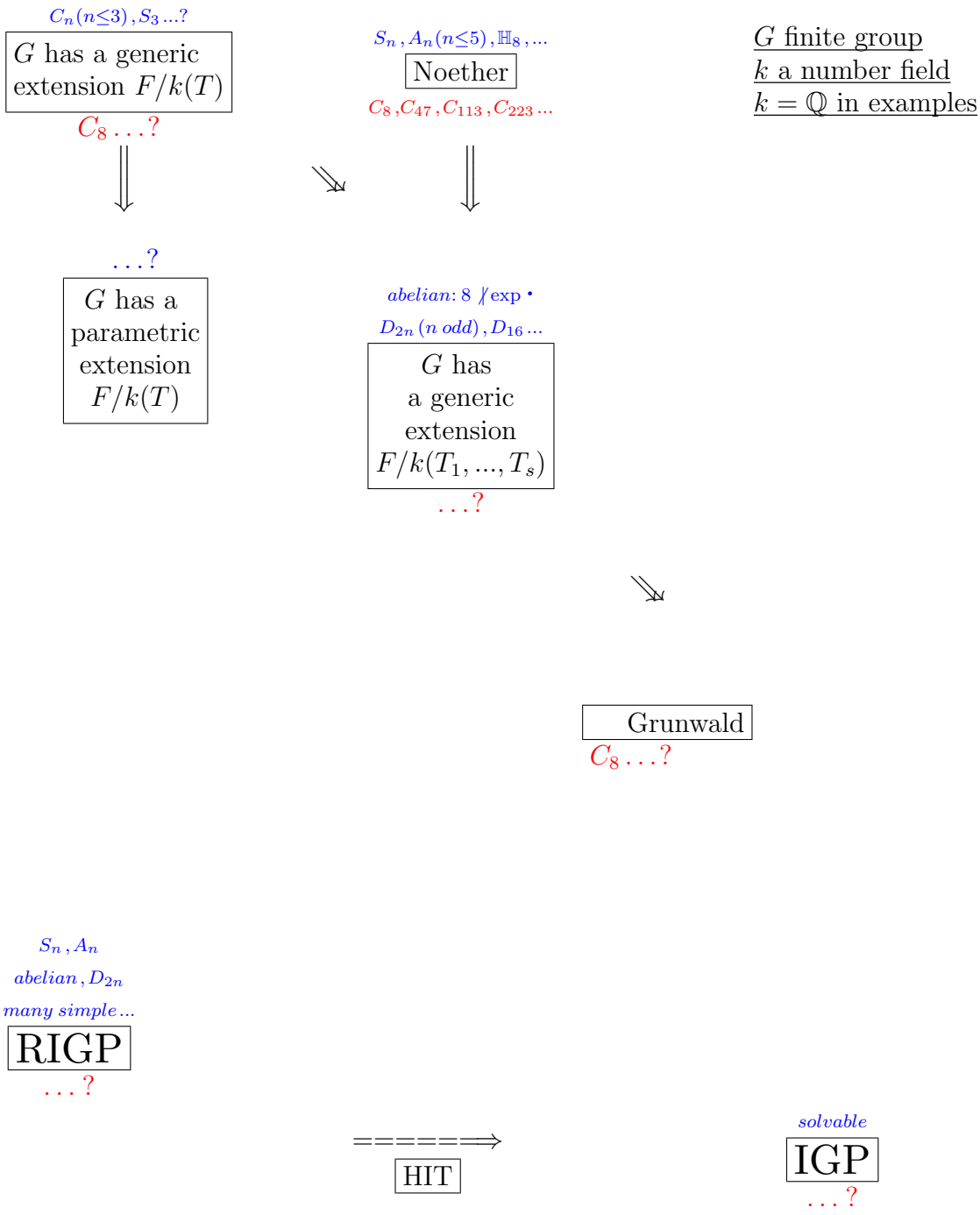
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*abelian*: $8 \nmid \exp \cdot$
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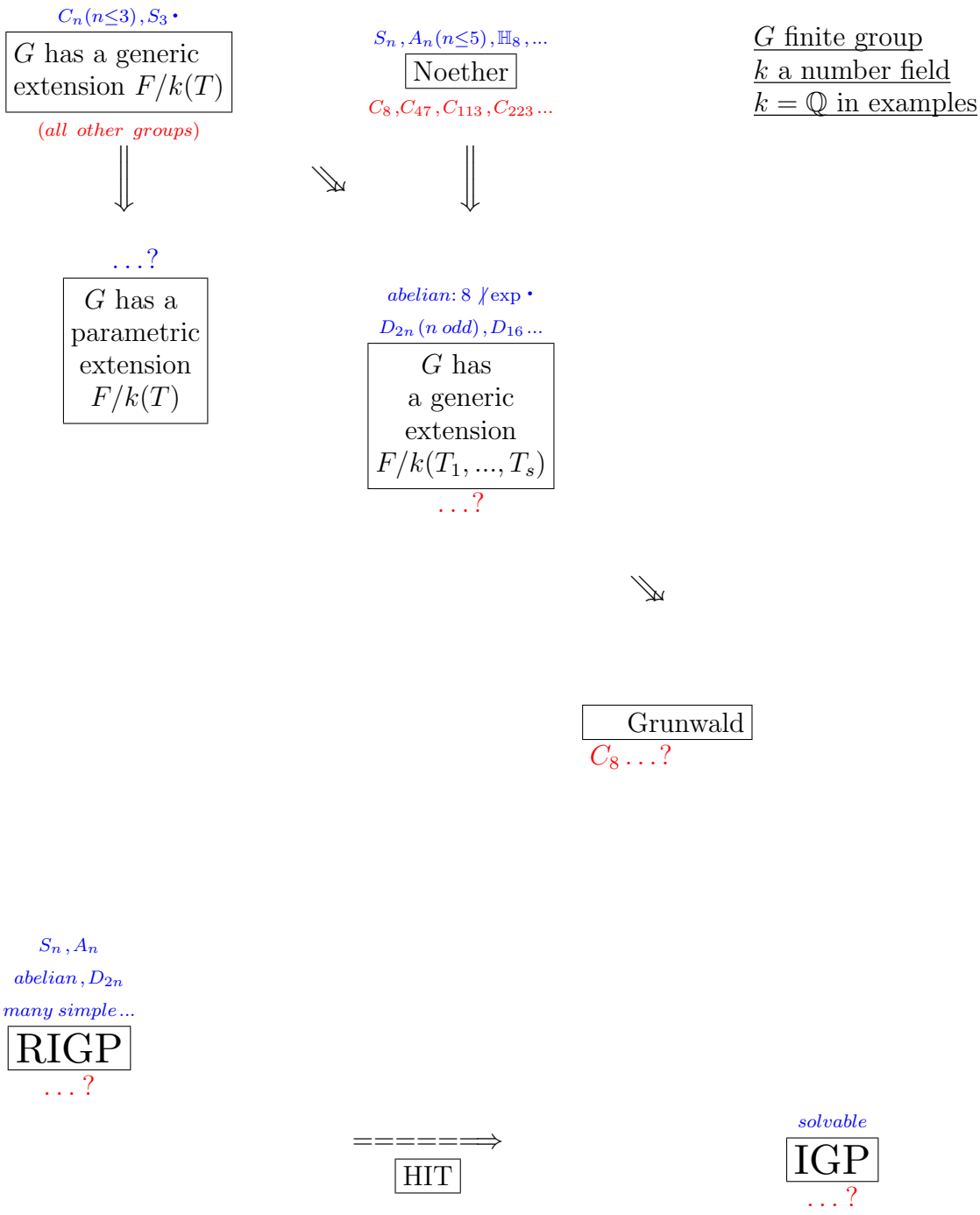
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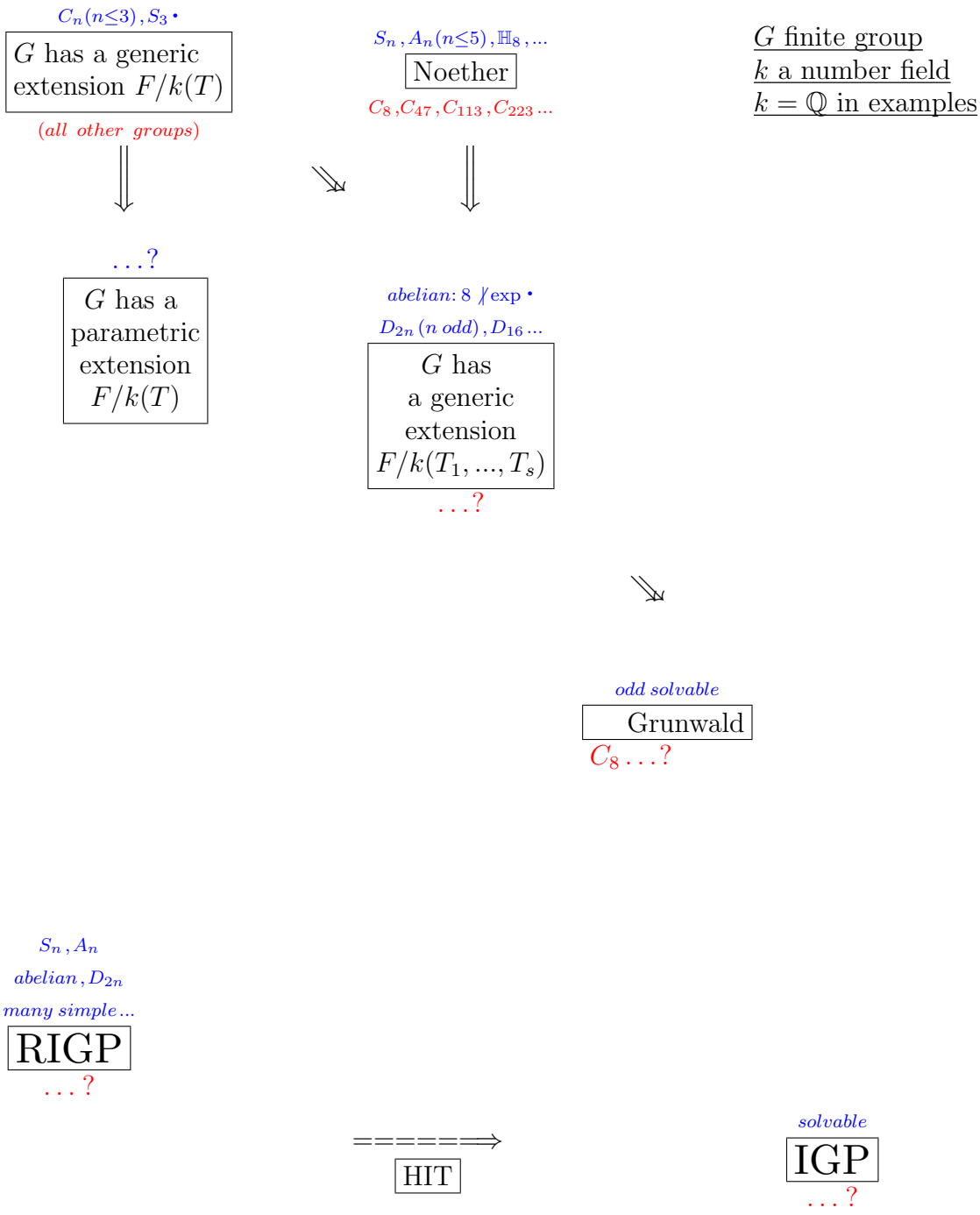
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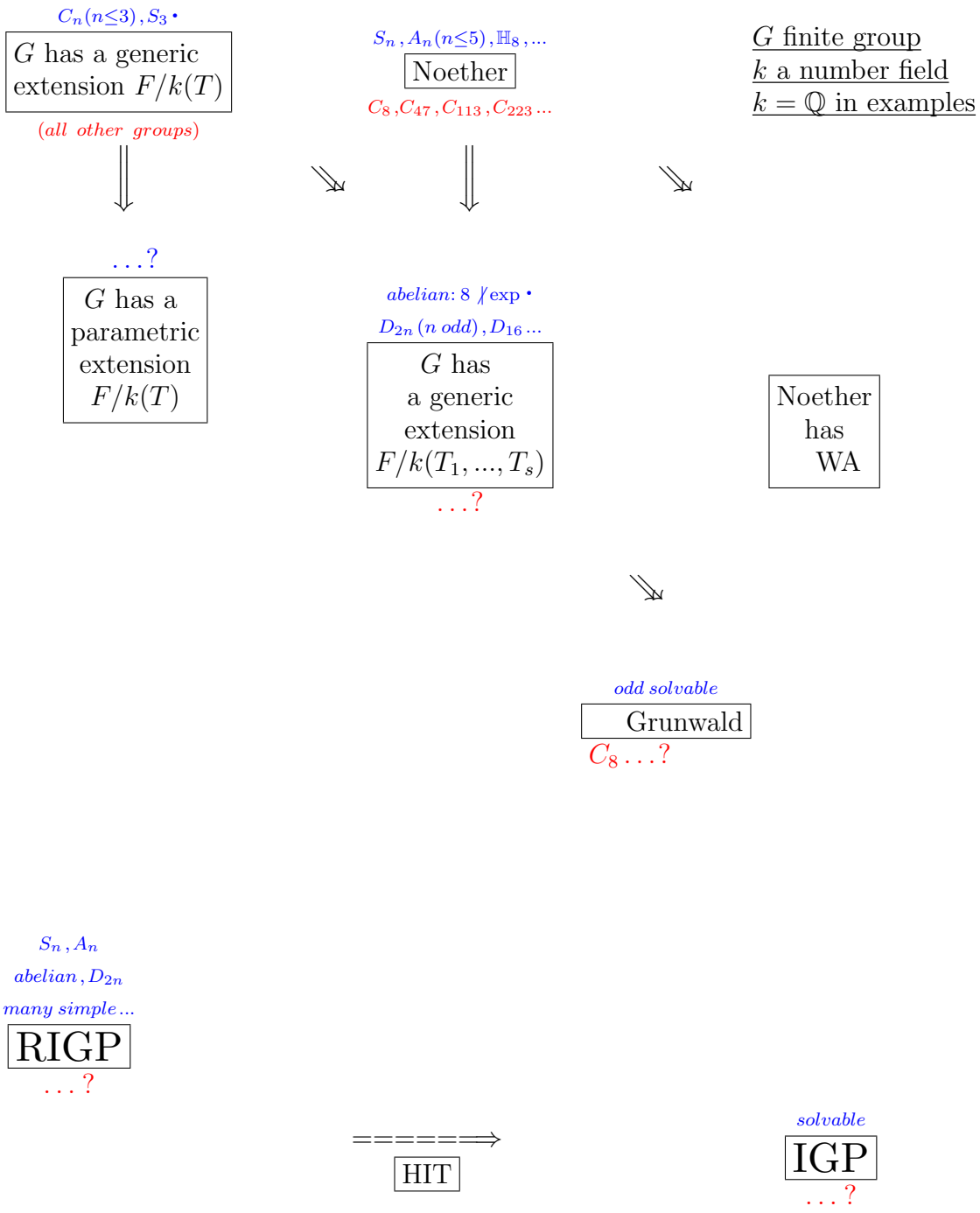
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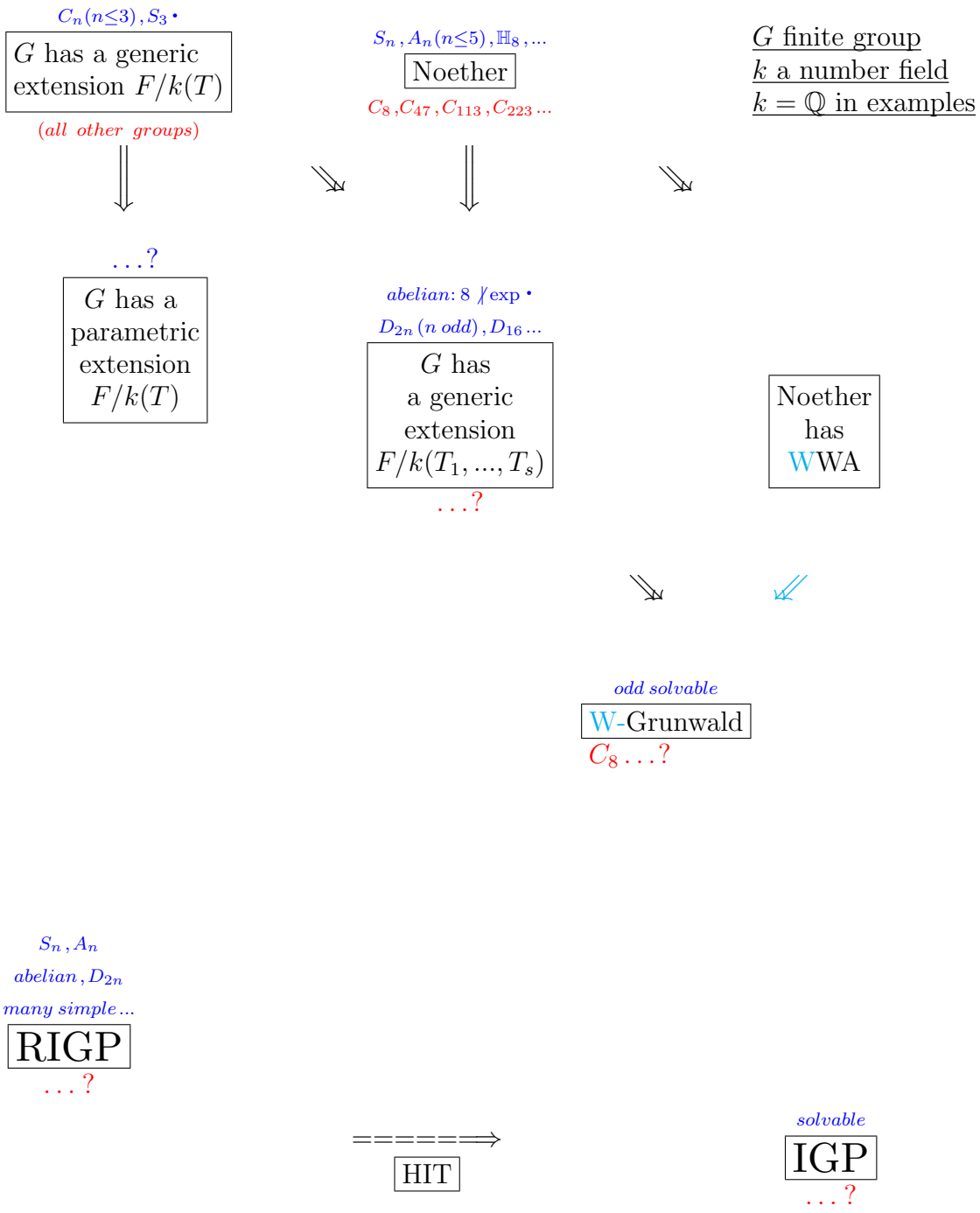
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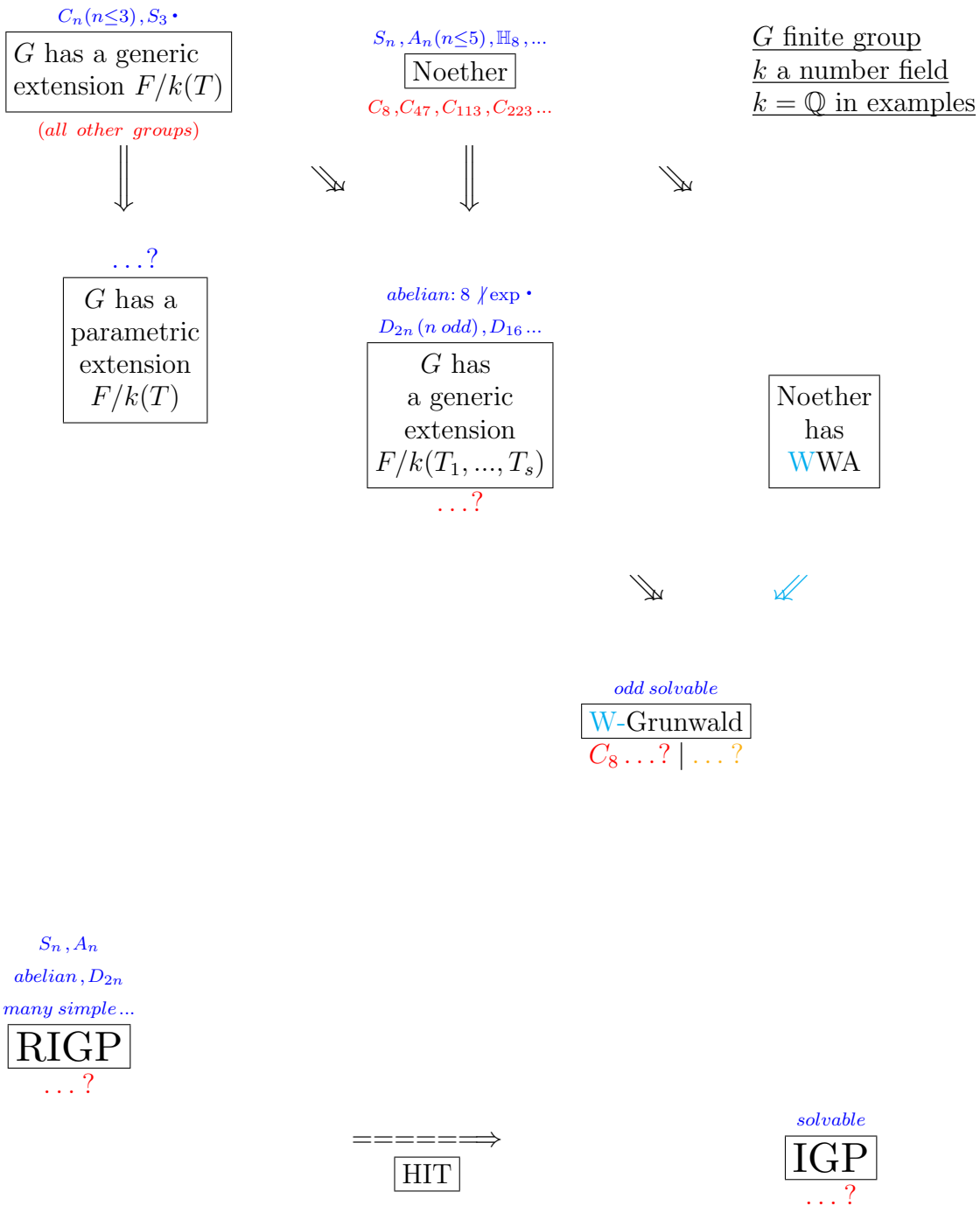
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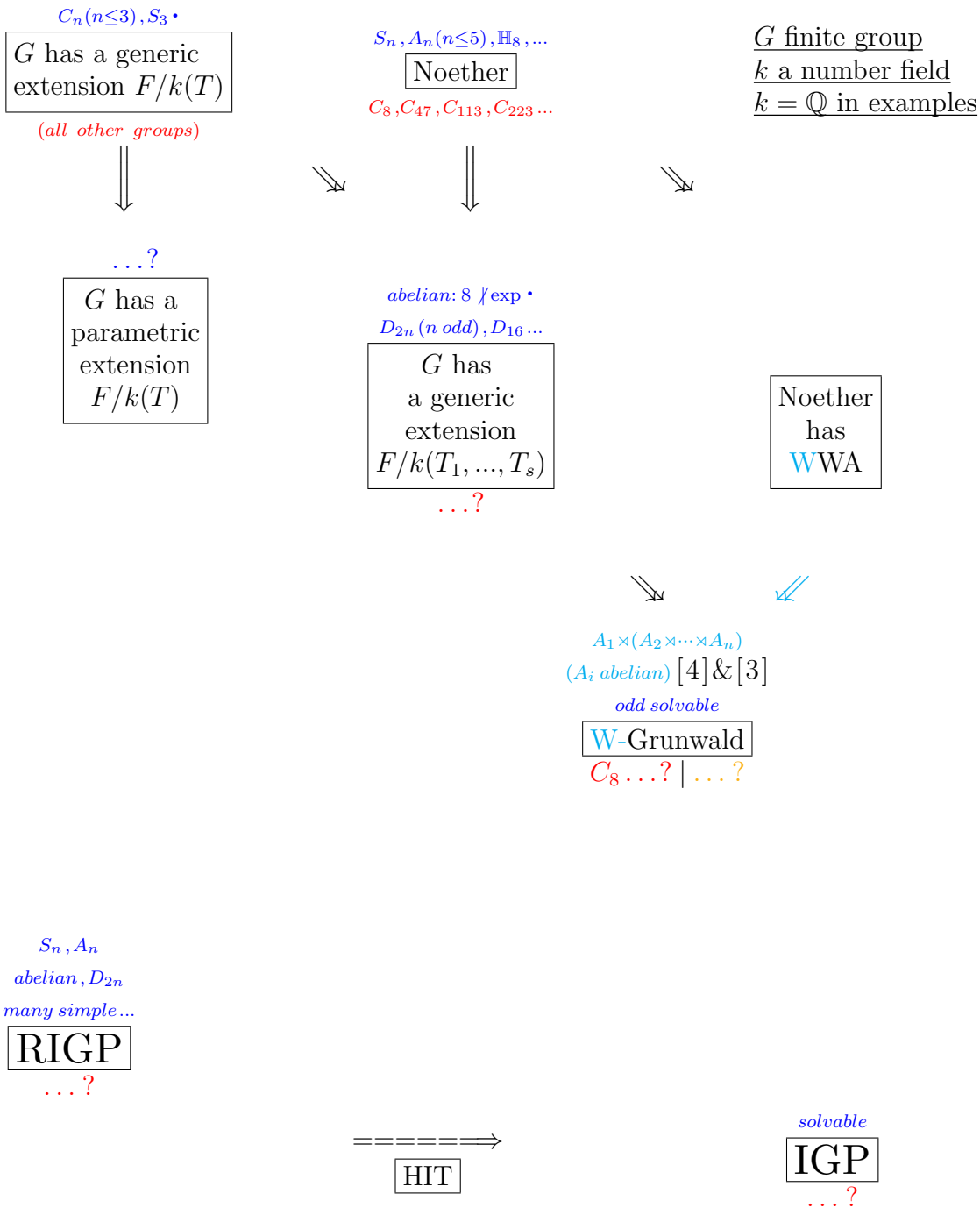
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$A_1 \rtimes (A_2 \rtimes \cdots \rtimes A_n)$
($A_i$ abelian) [4] & [3]
odd solvable
W-Grunwald
$C_8$ . . . ? | . . . ?
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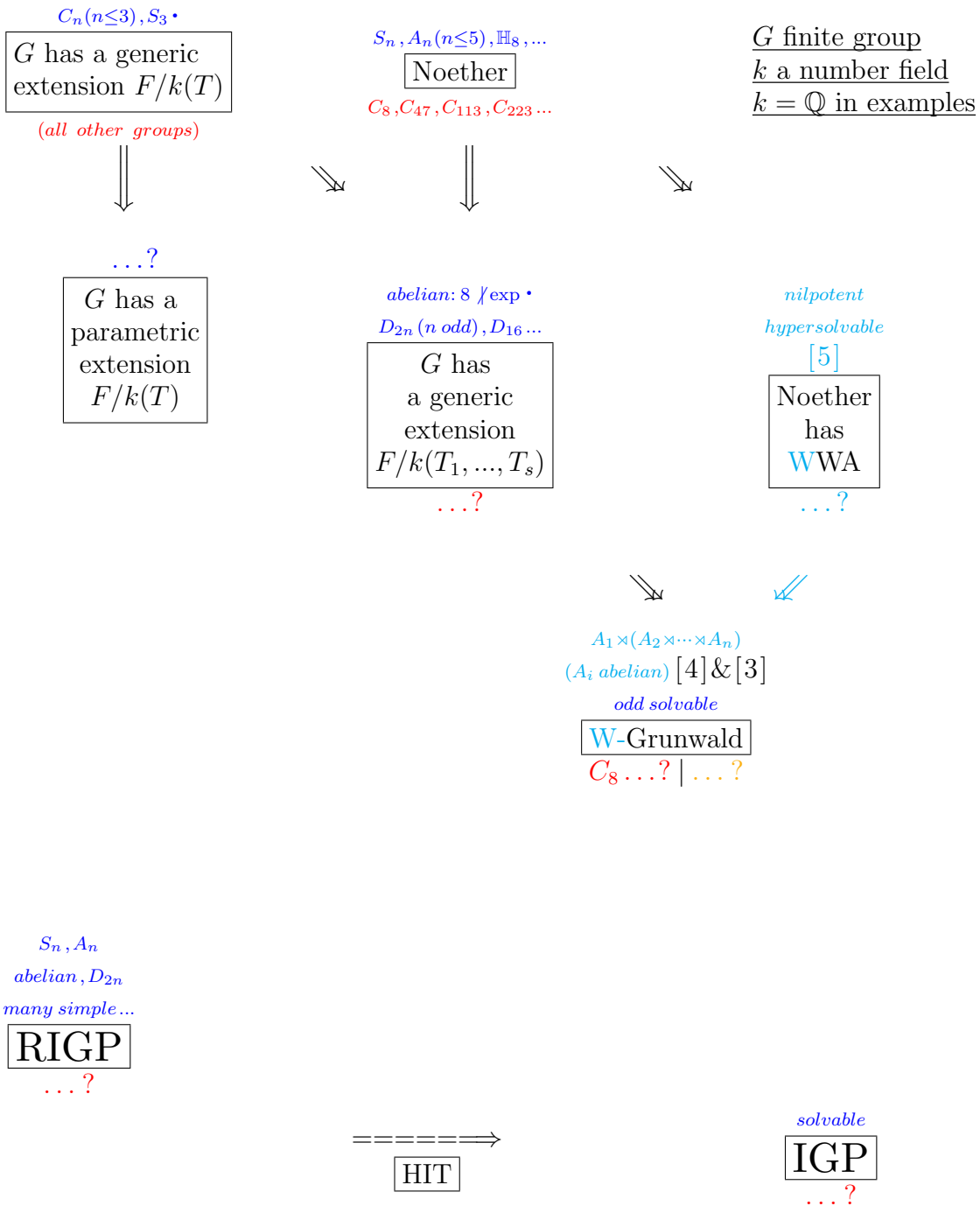
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. . . ?
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nilpotent
hypersolvable
[5]
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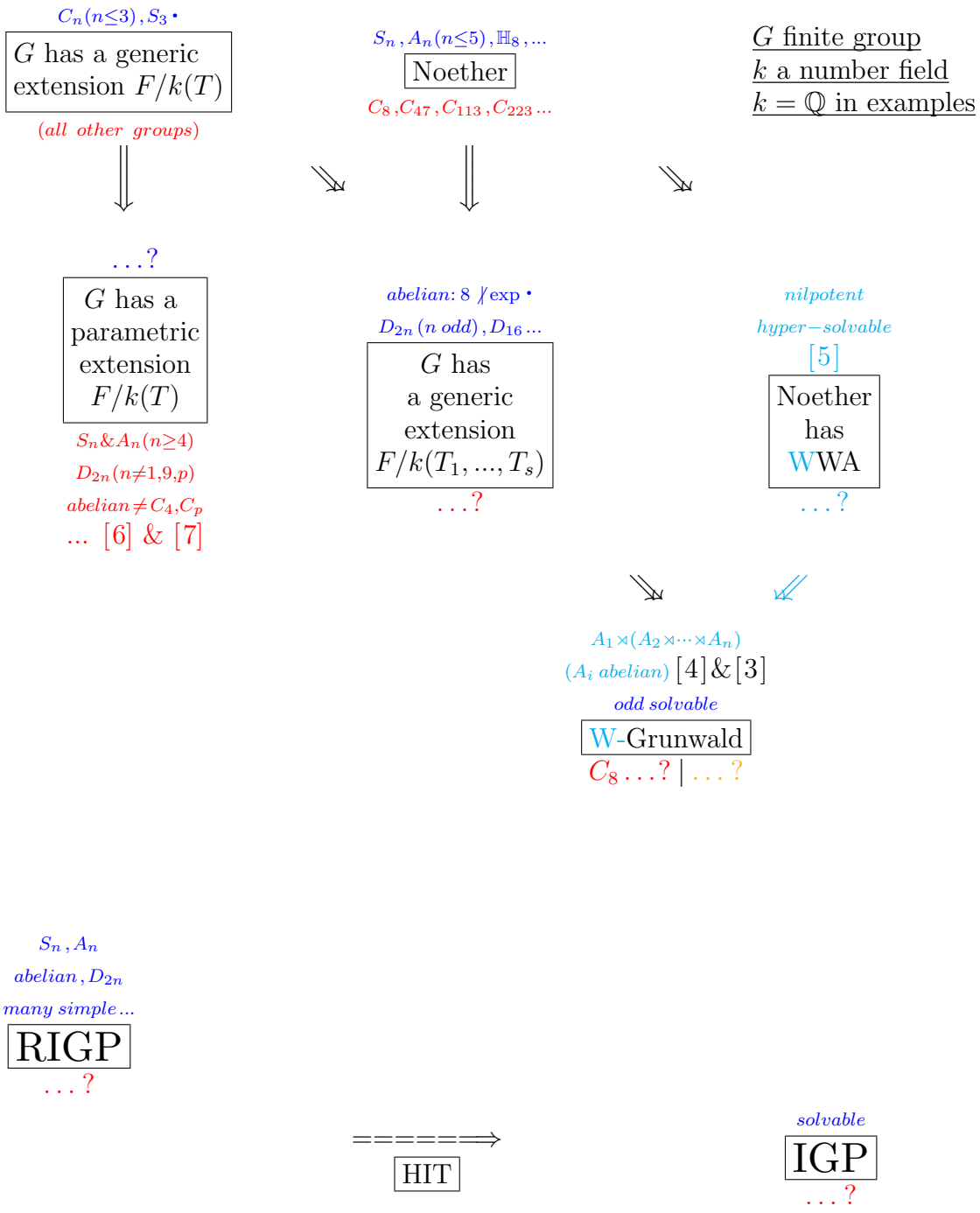
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$S_n \& A_n (n \geq 4)$
$D_{2n} (n \neq 1,9,p)$
abelian $\neq C_4, C_p$
... [6] & [7]
$S_n, A_n (n \leq 5), \mathbb{H}_8, ...$
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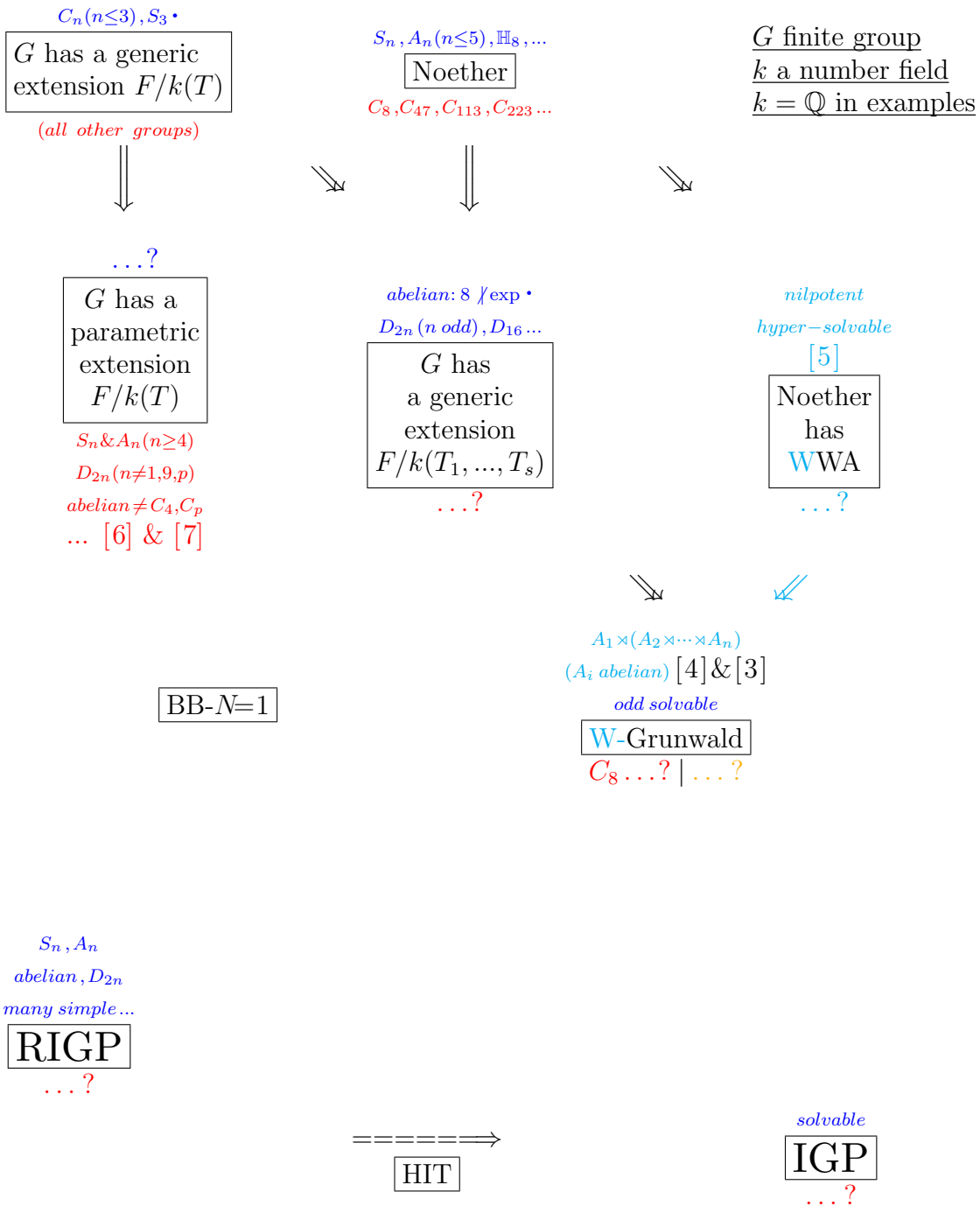
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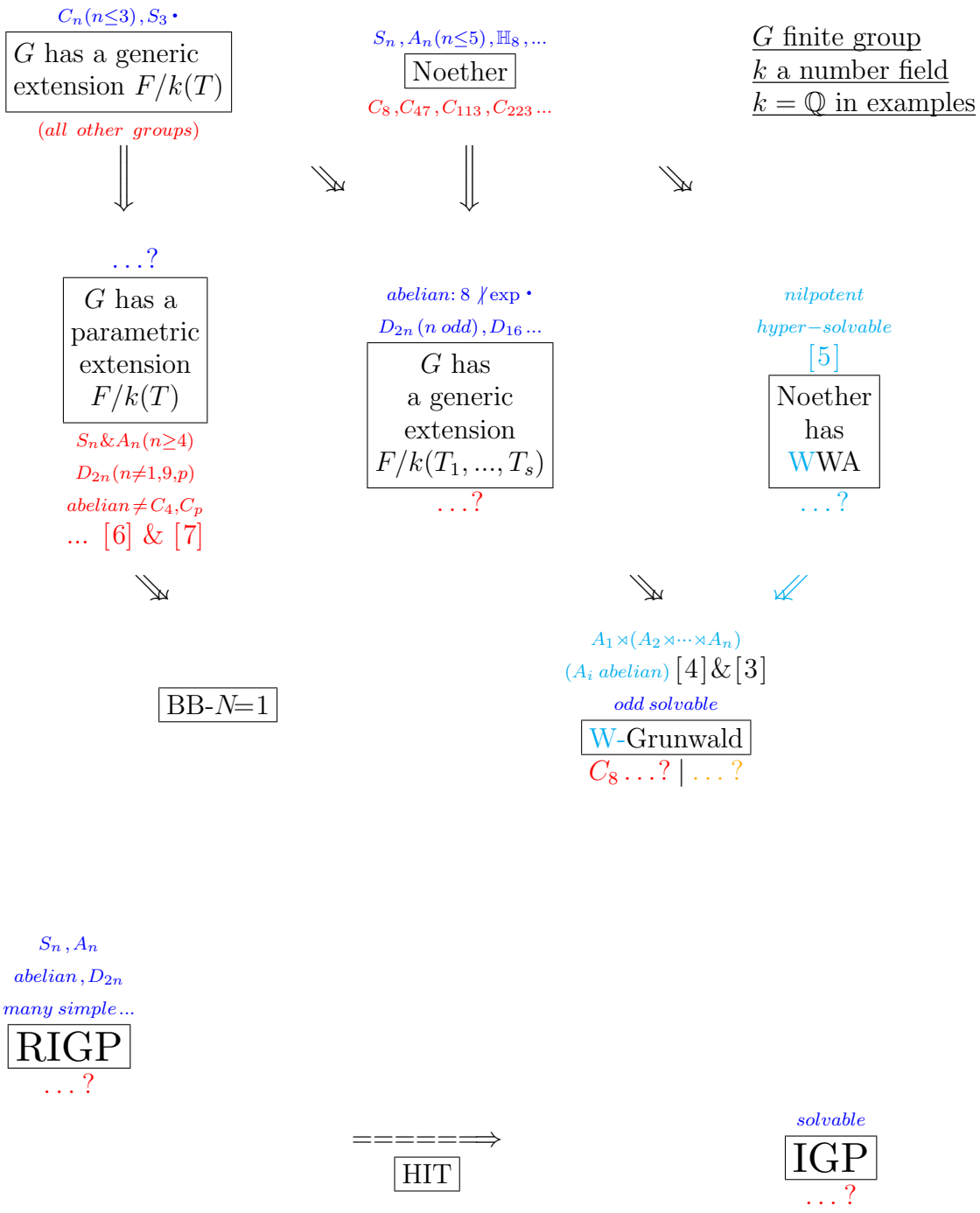
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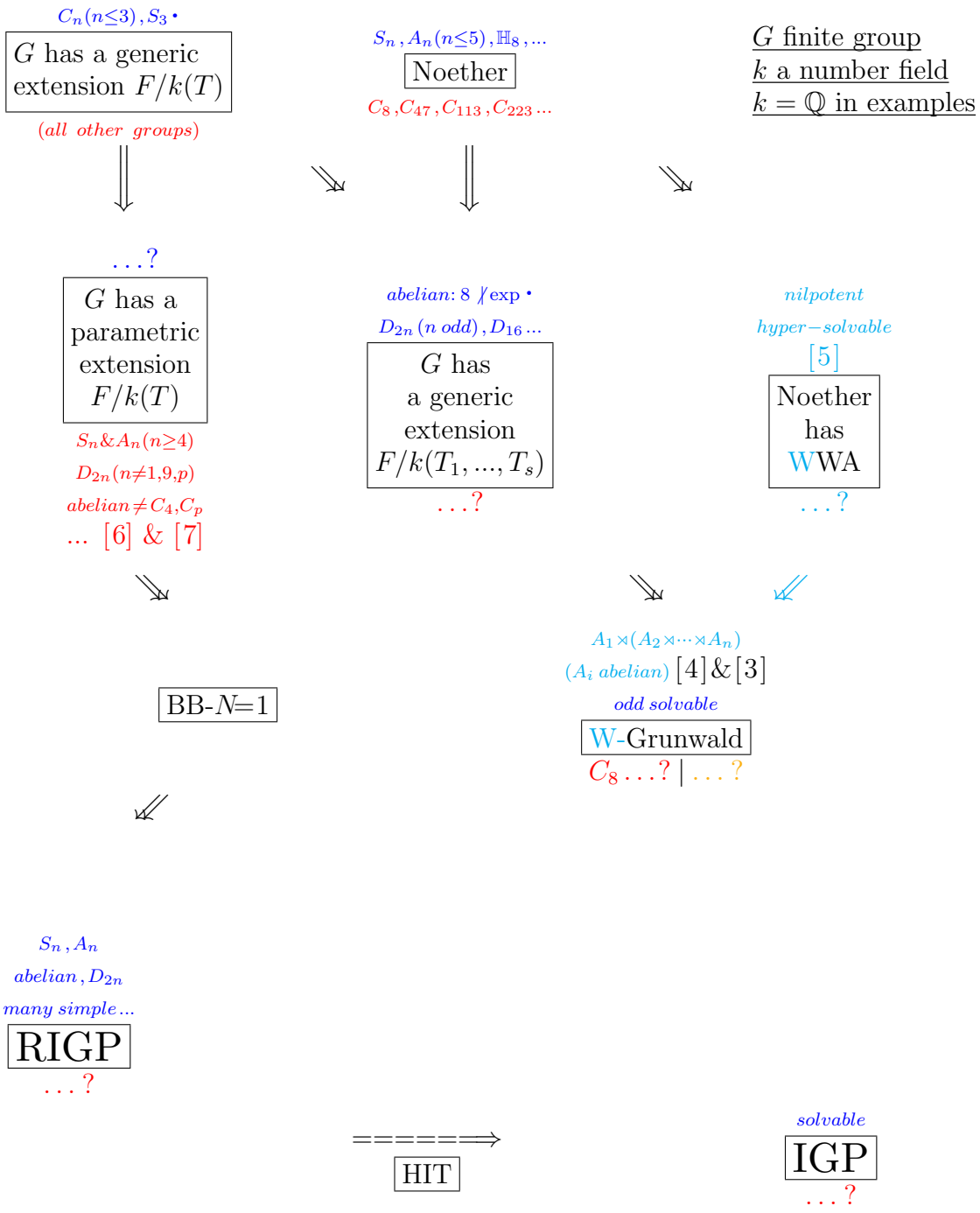
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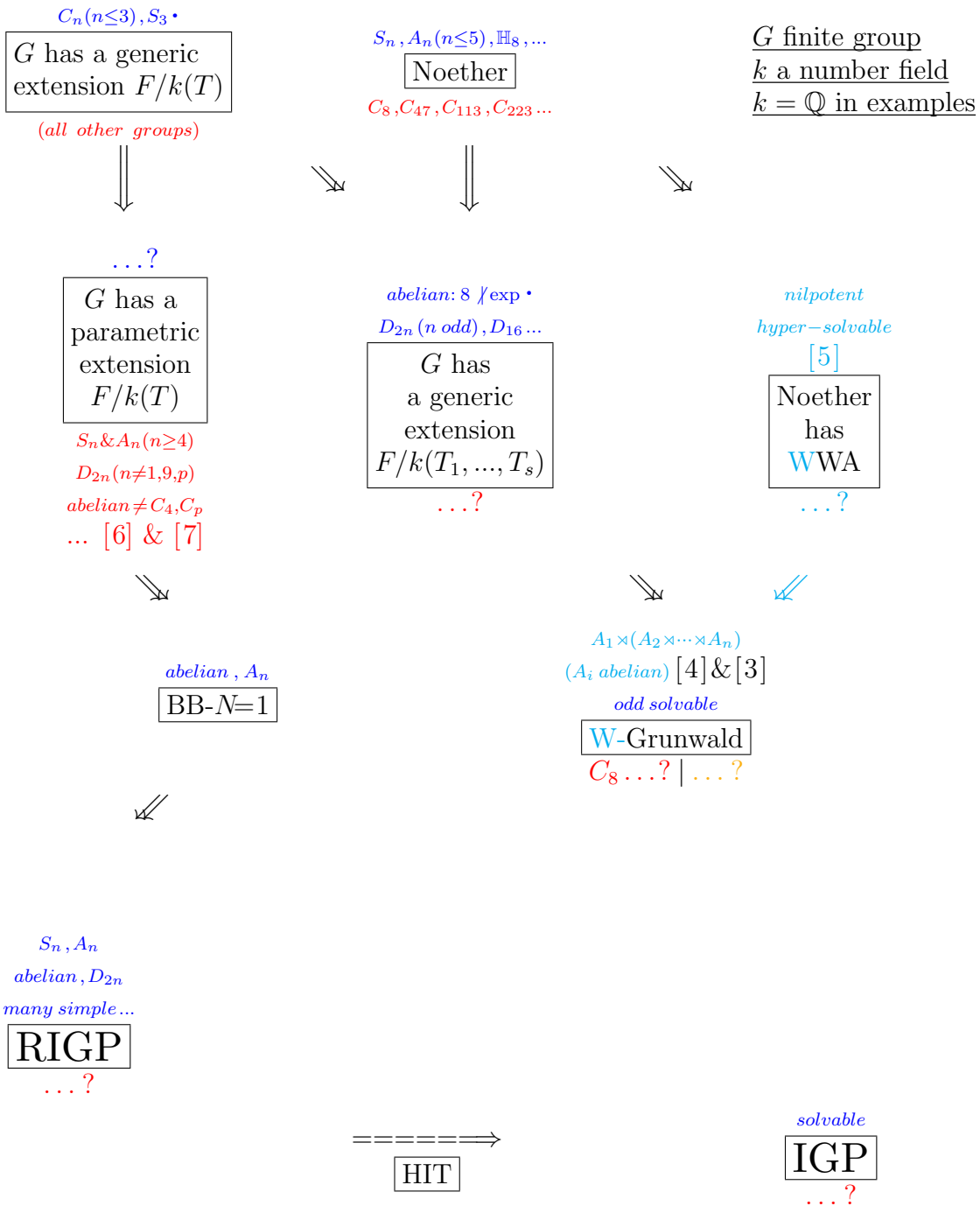
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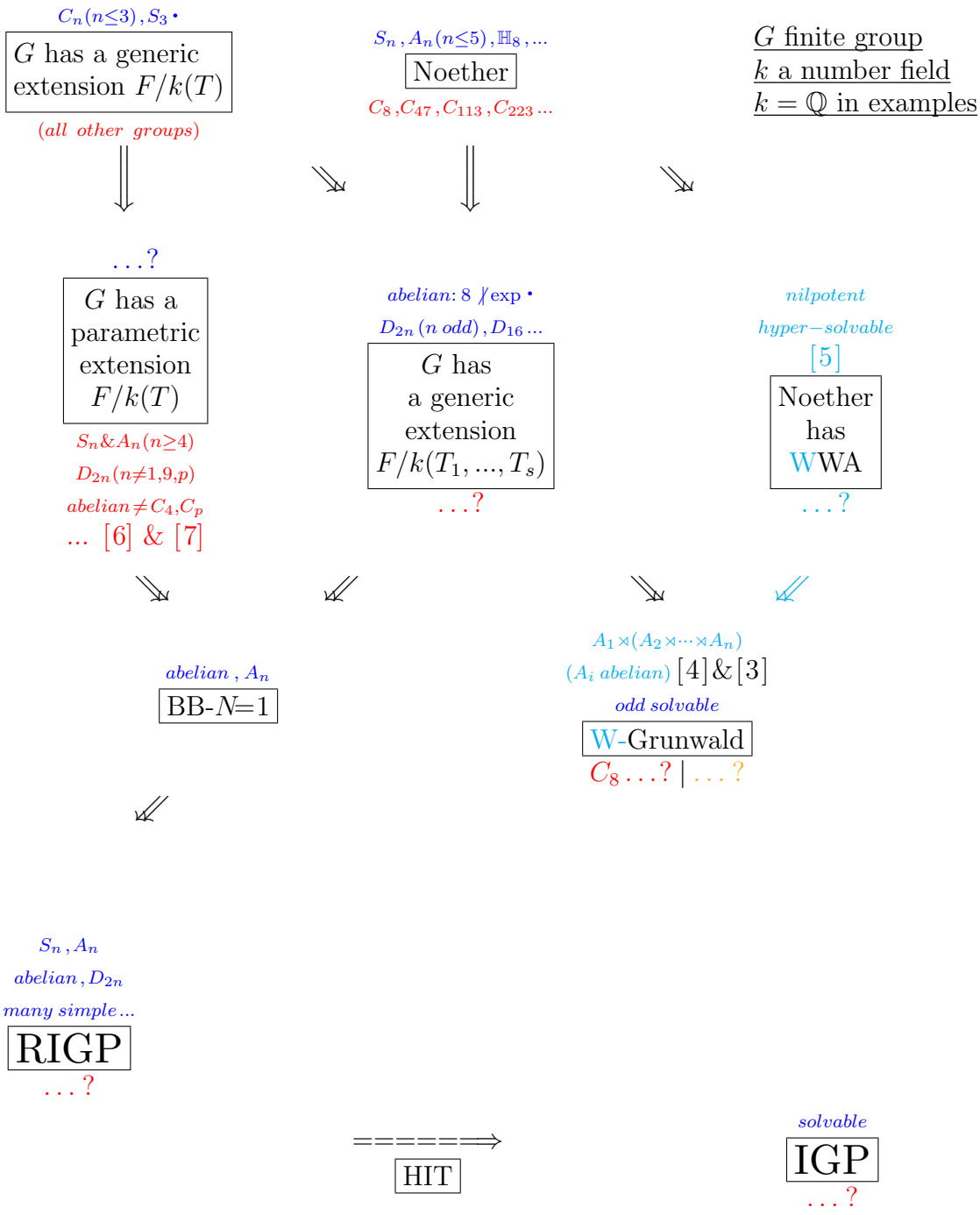
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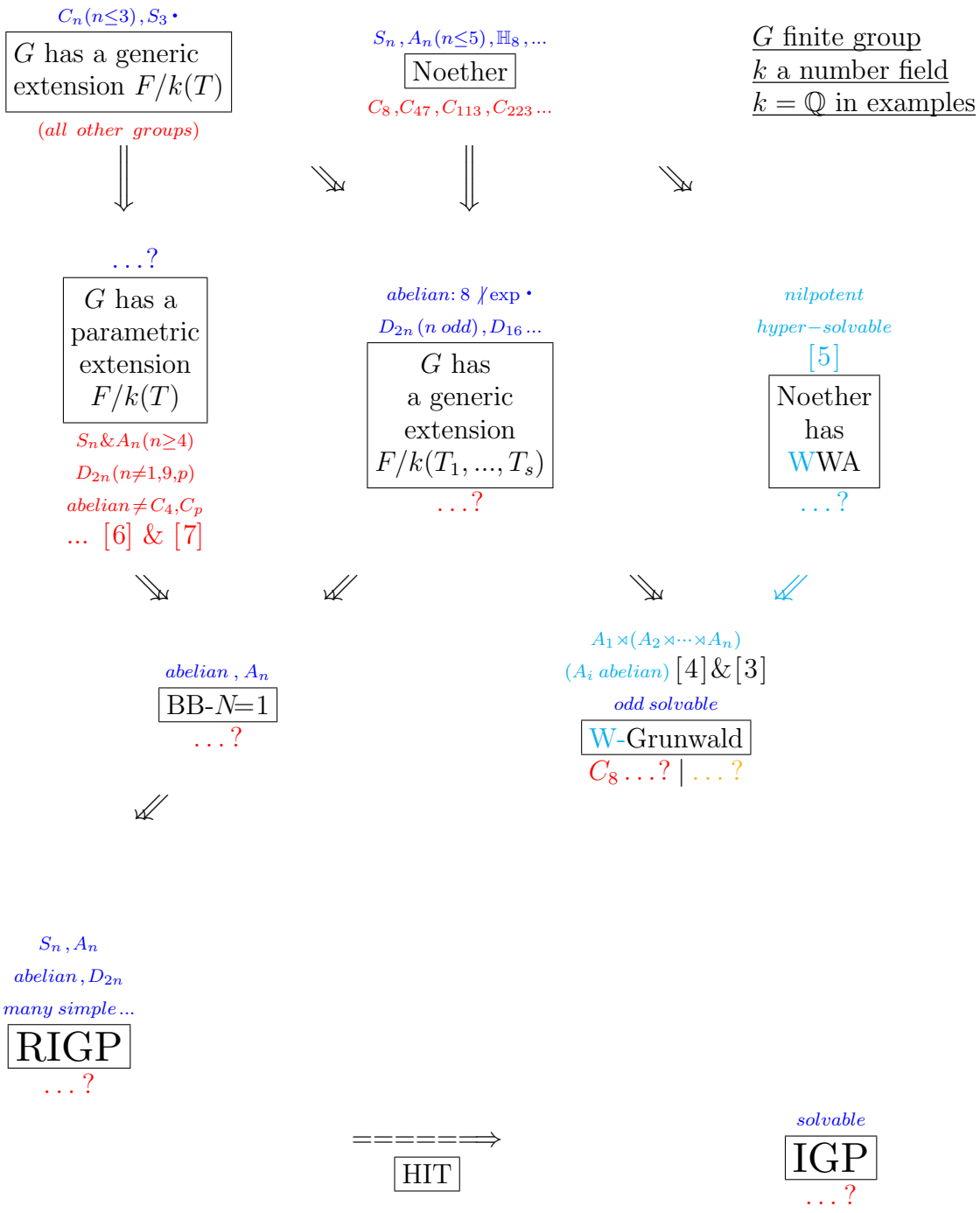
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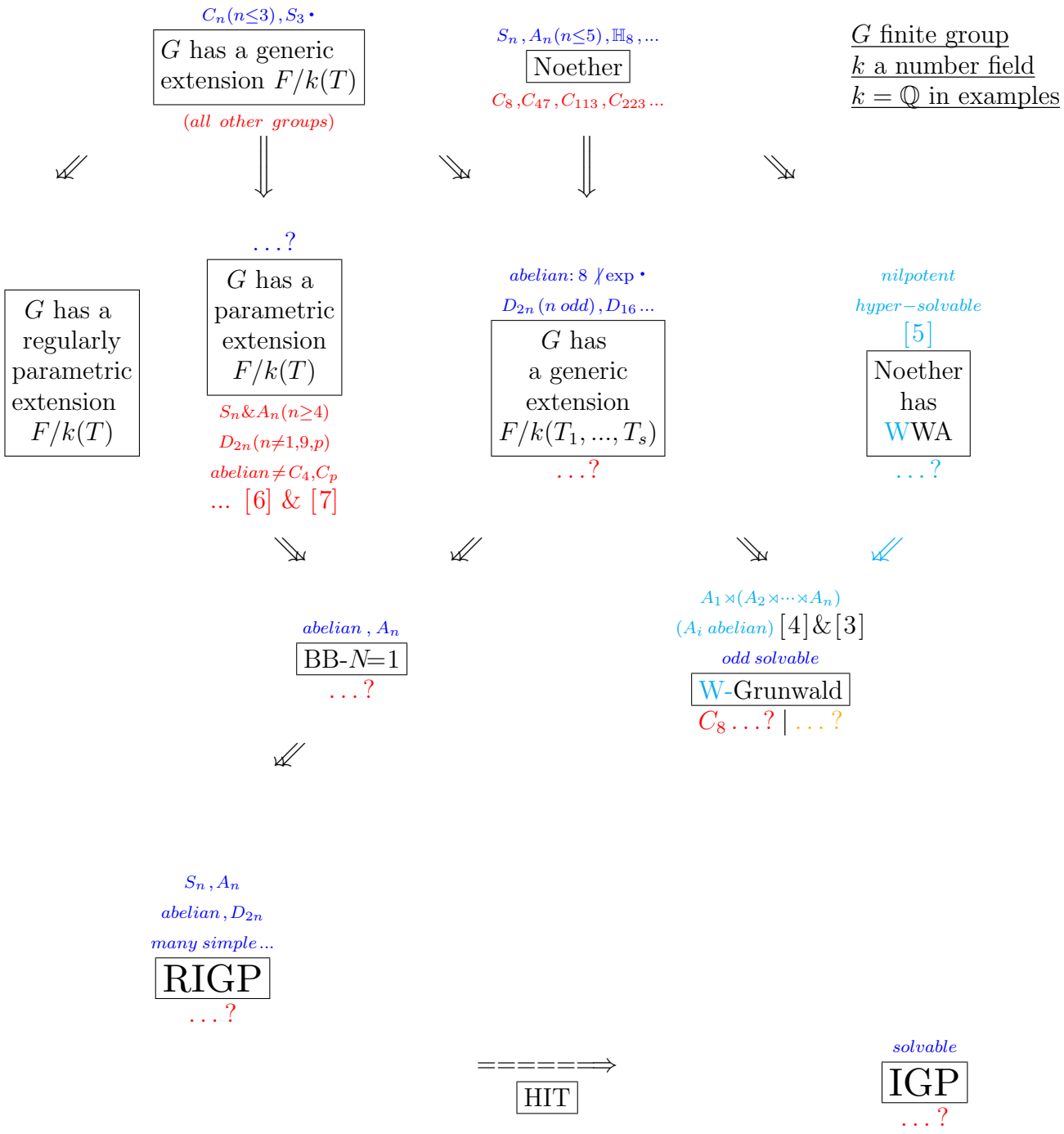
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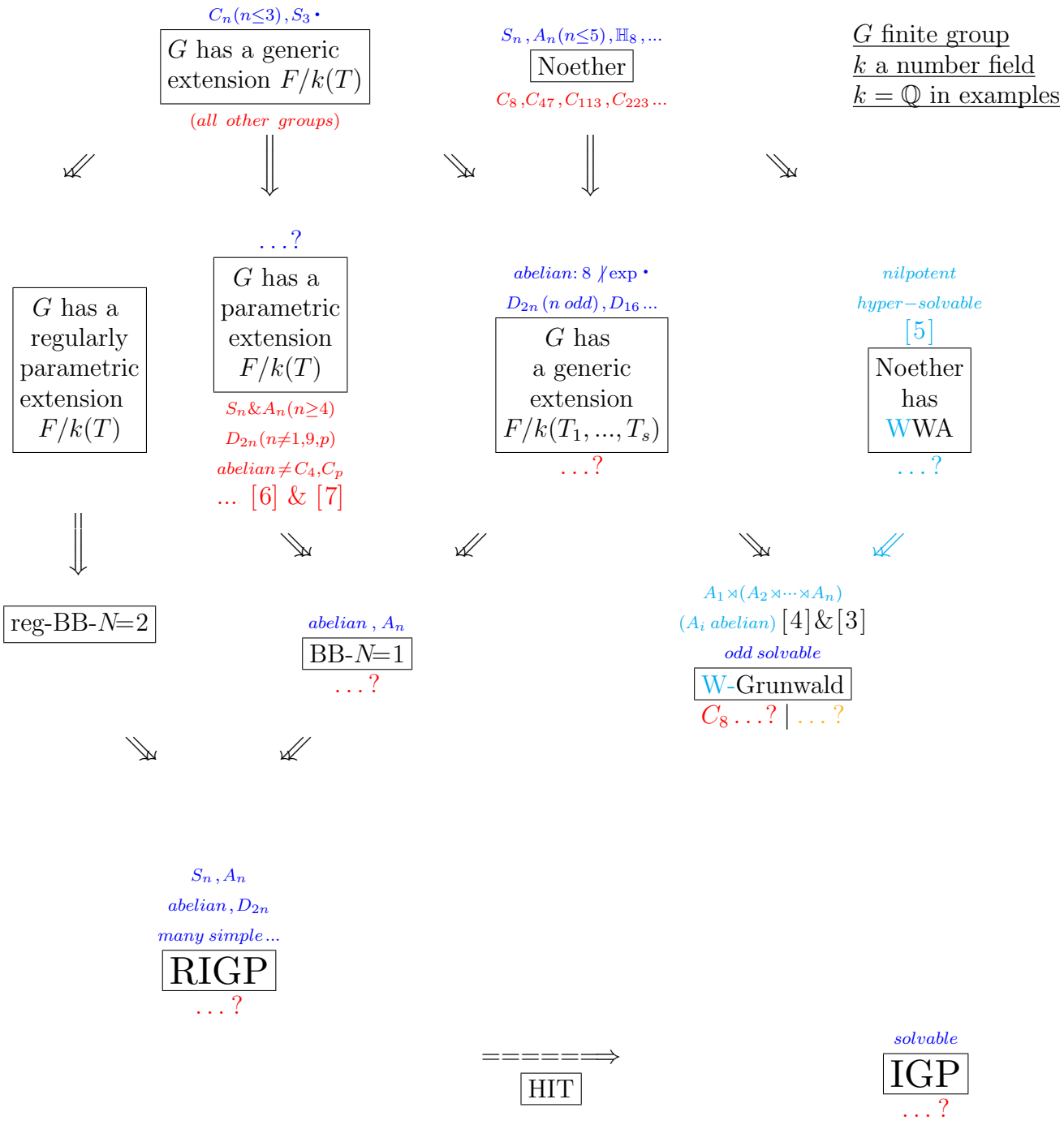
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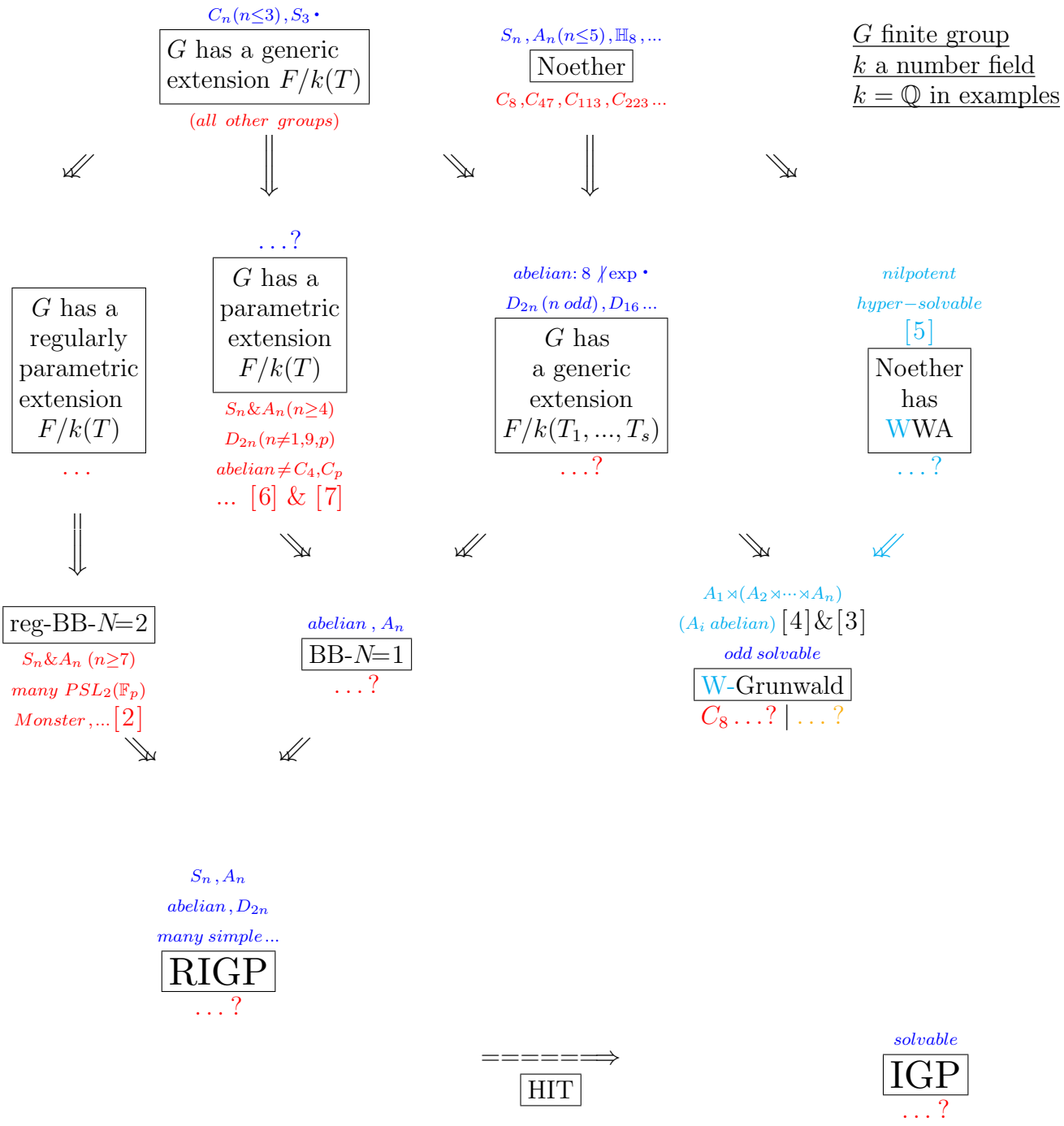
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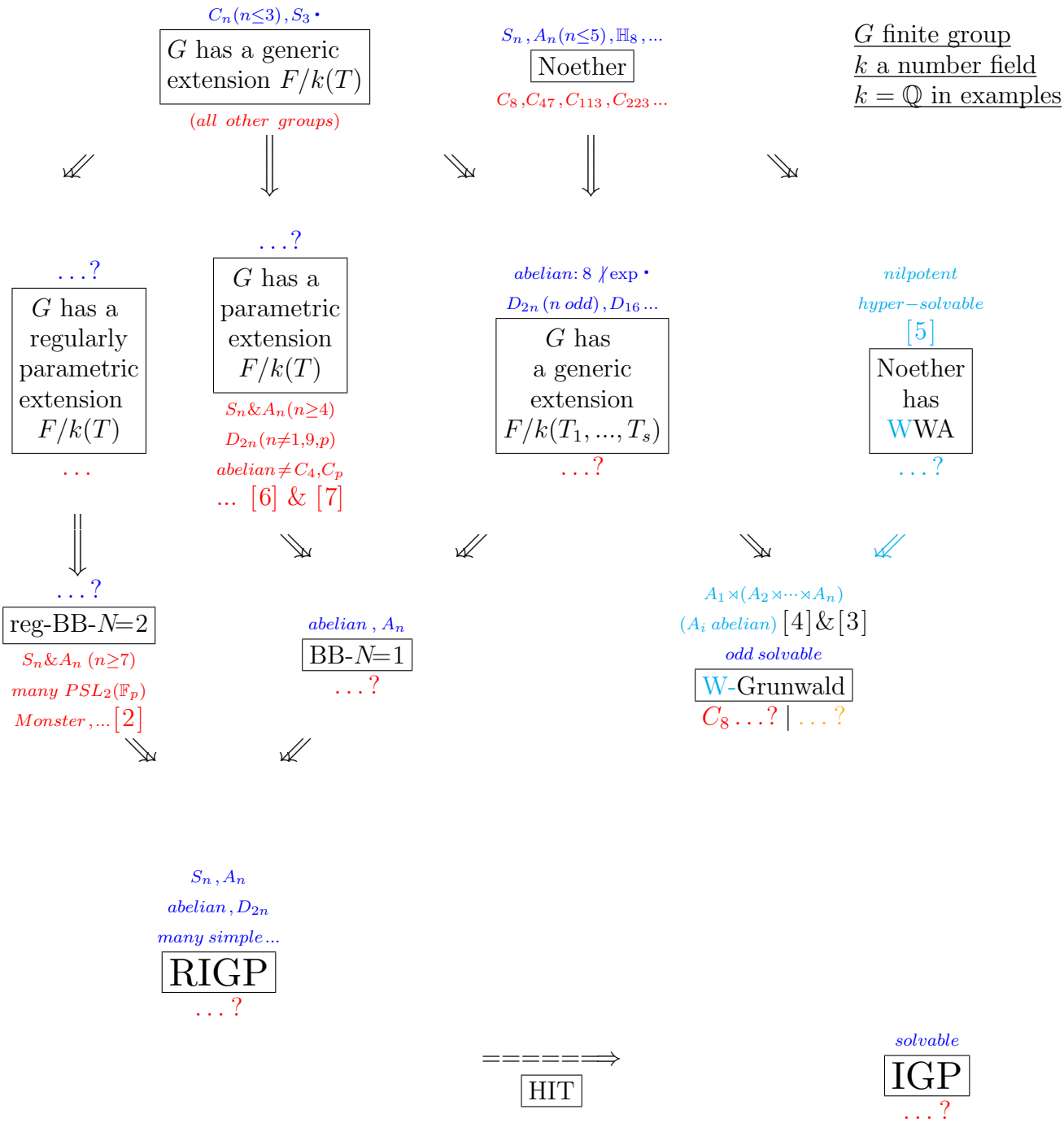
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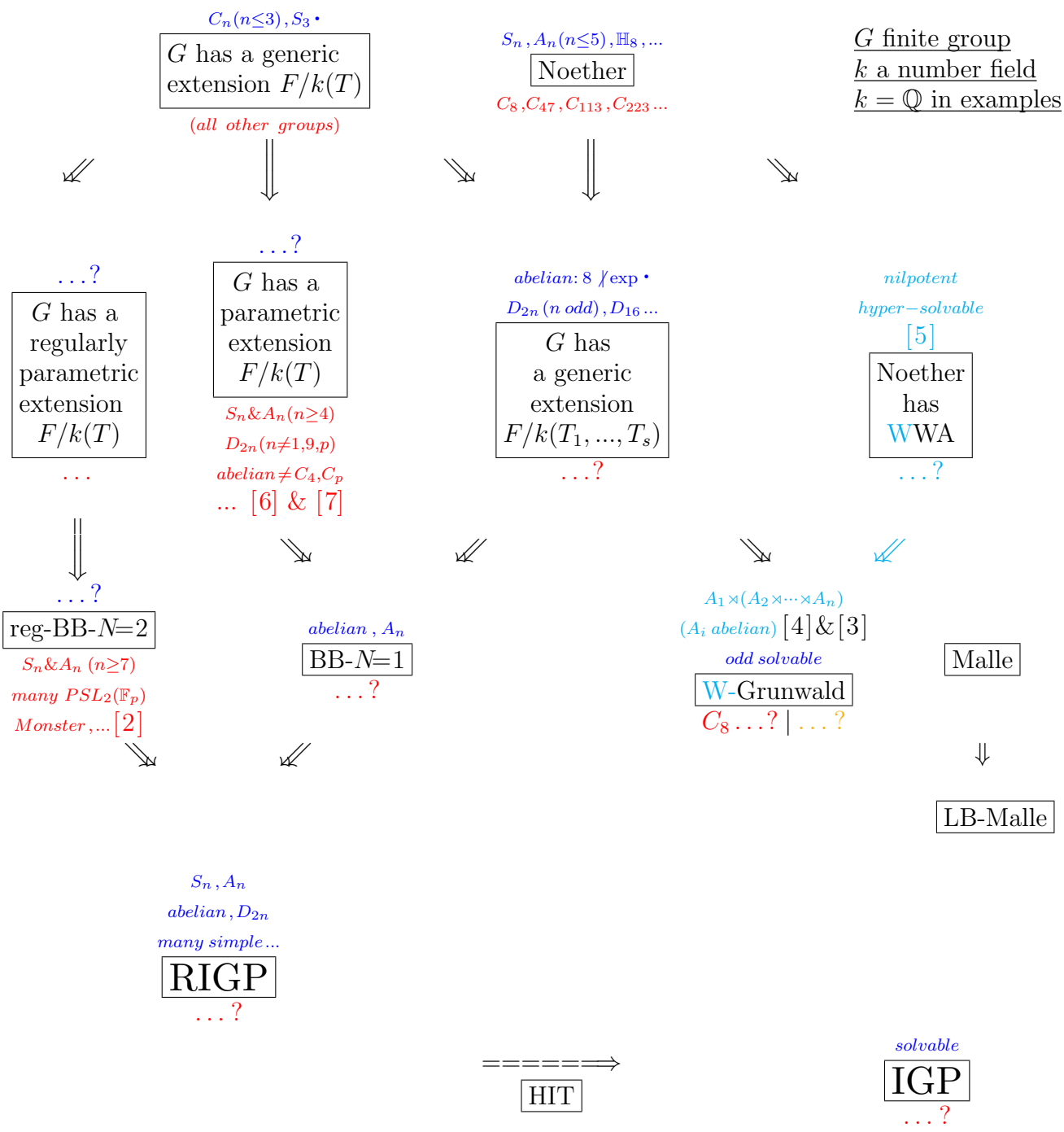
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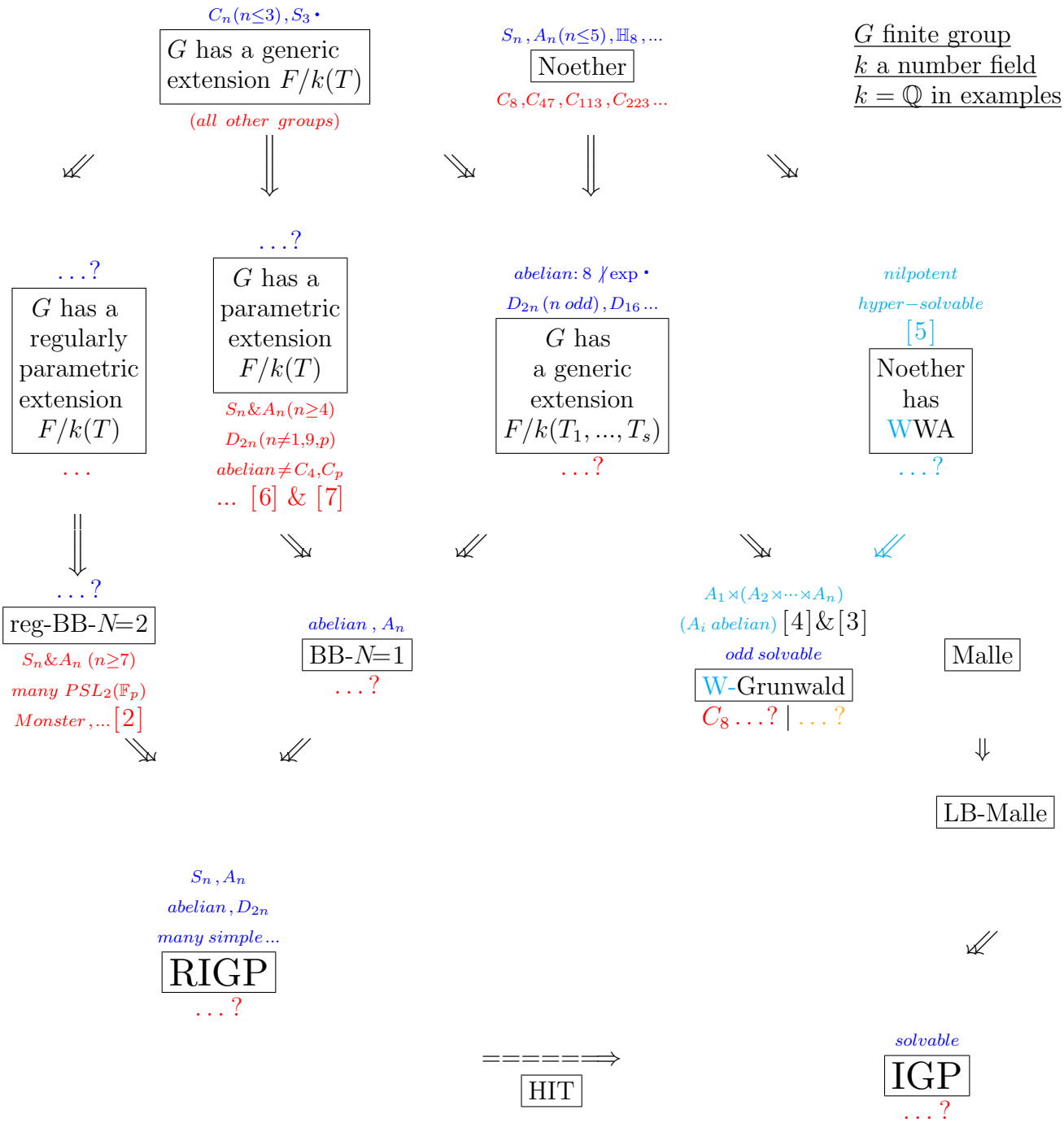
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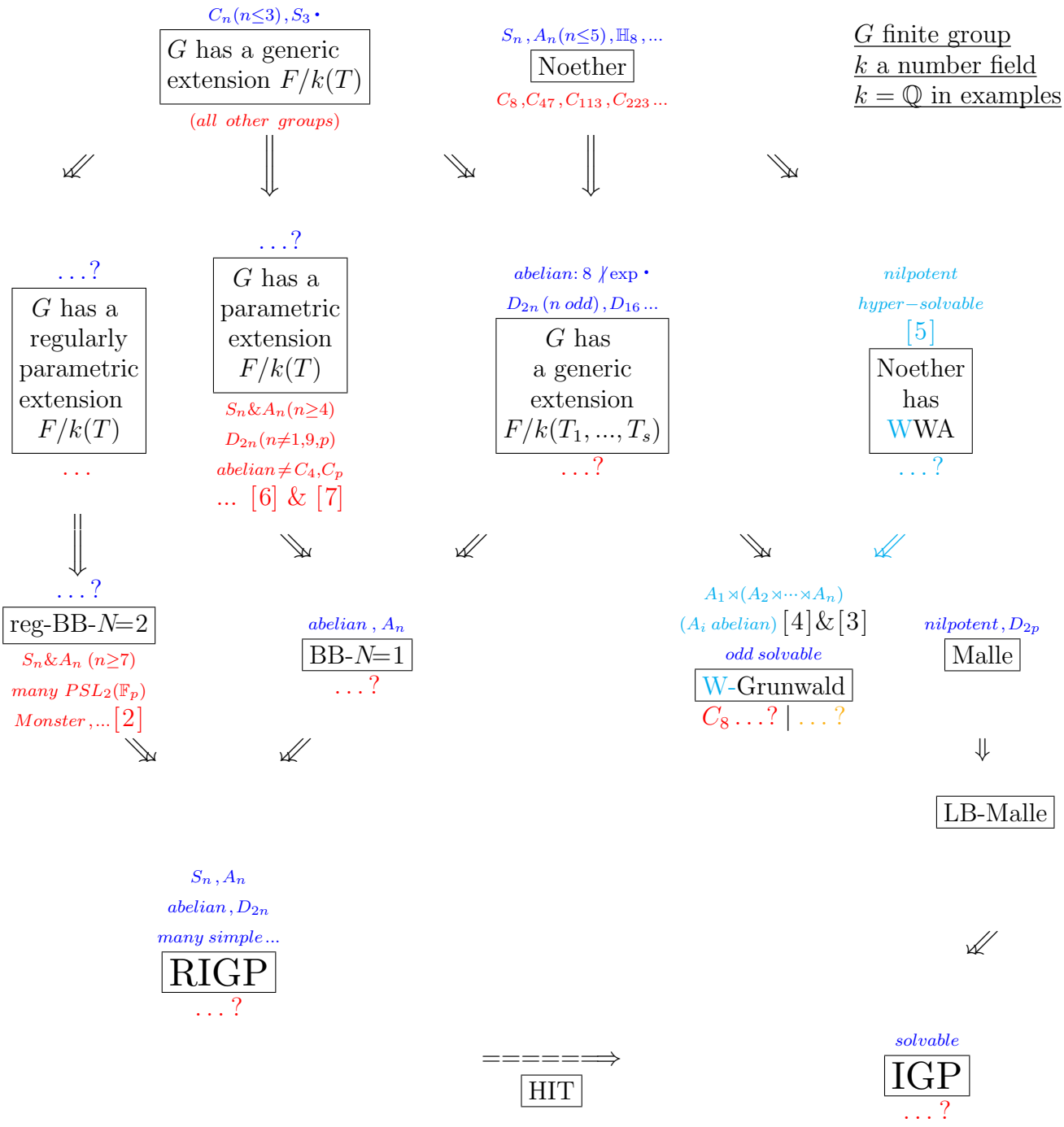
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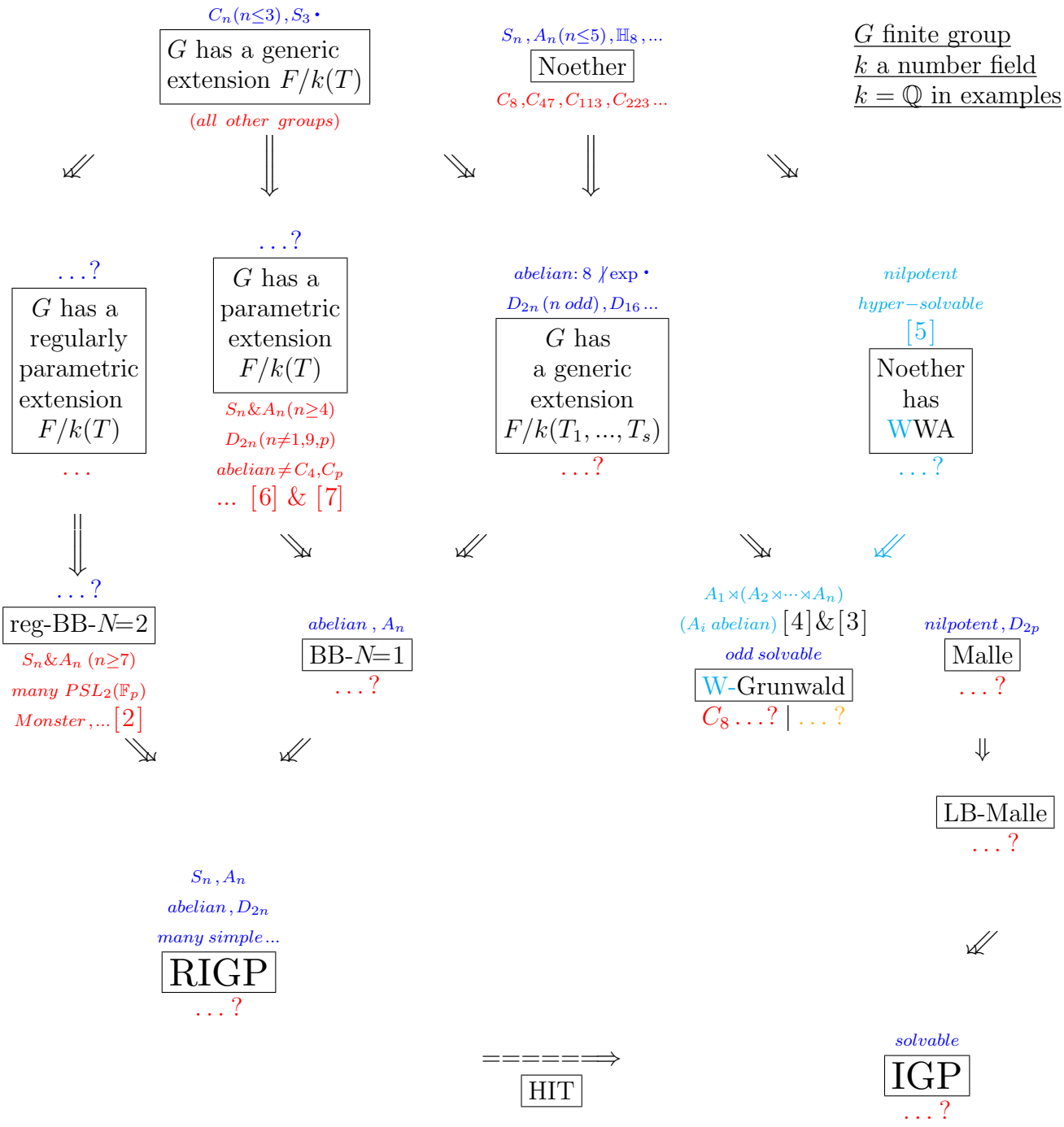
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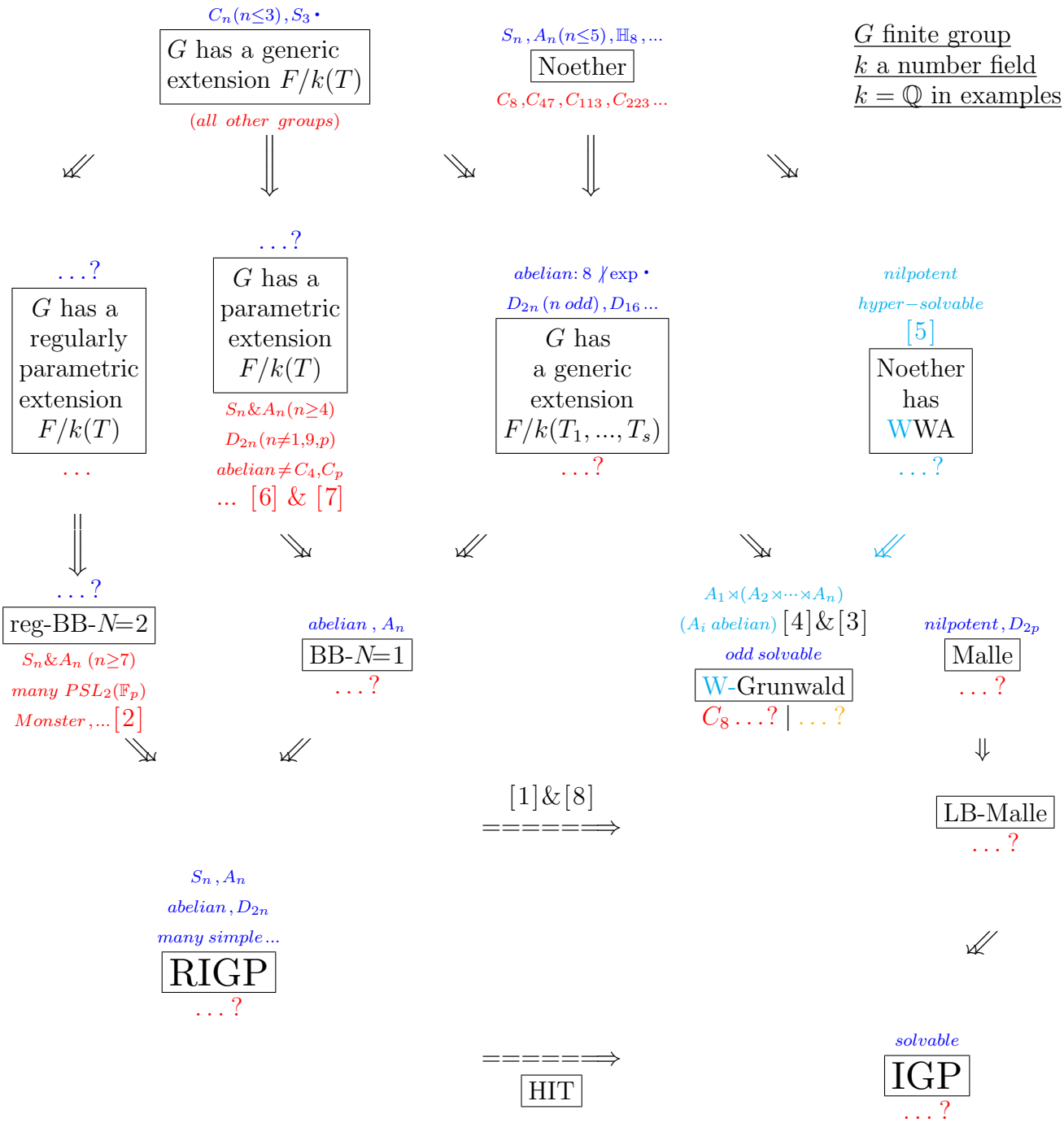
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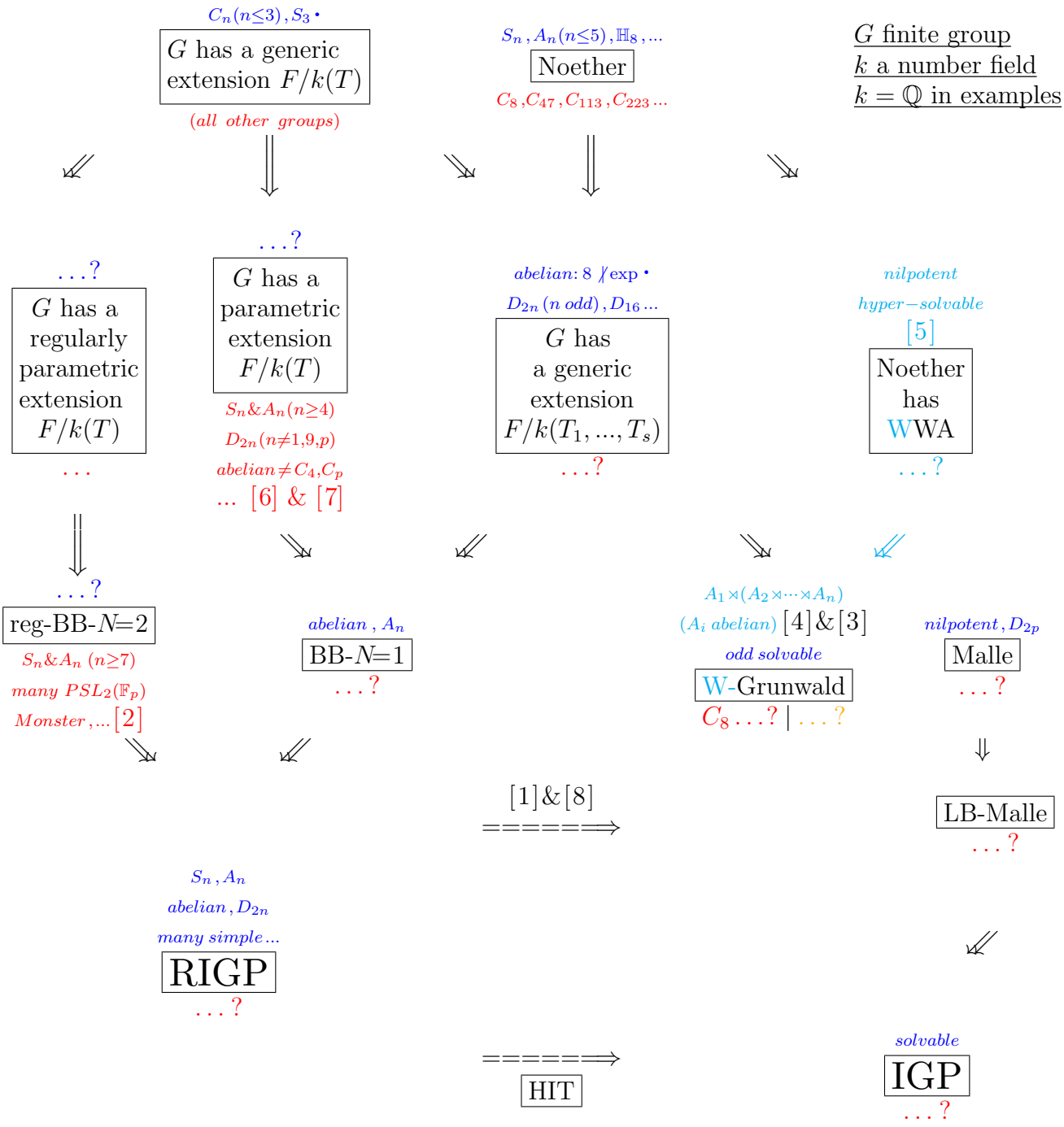
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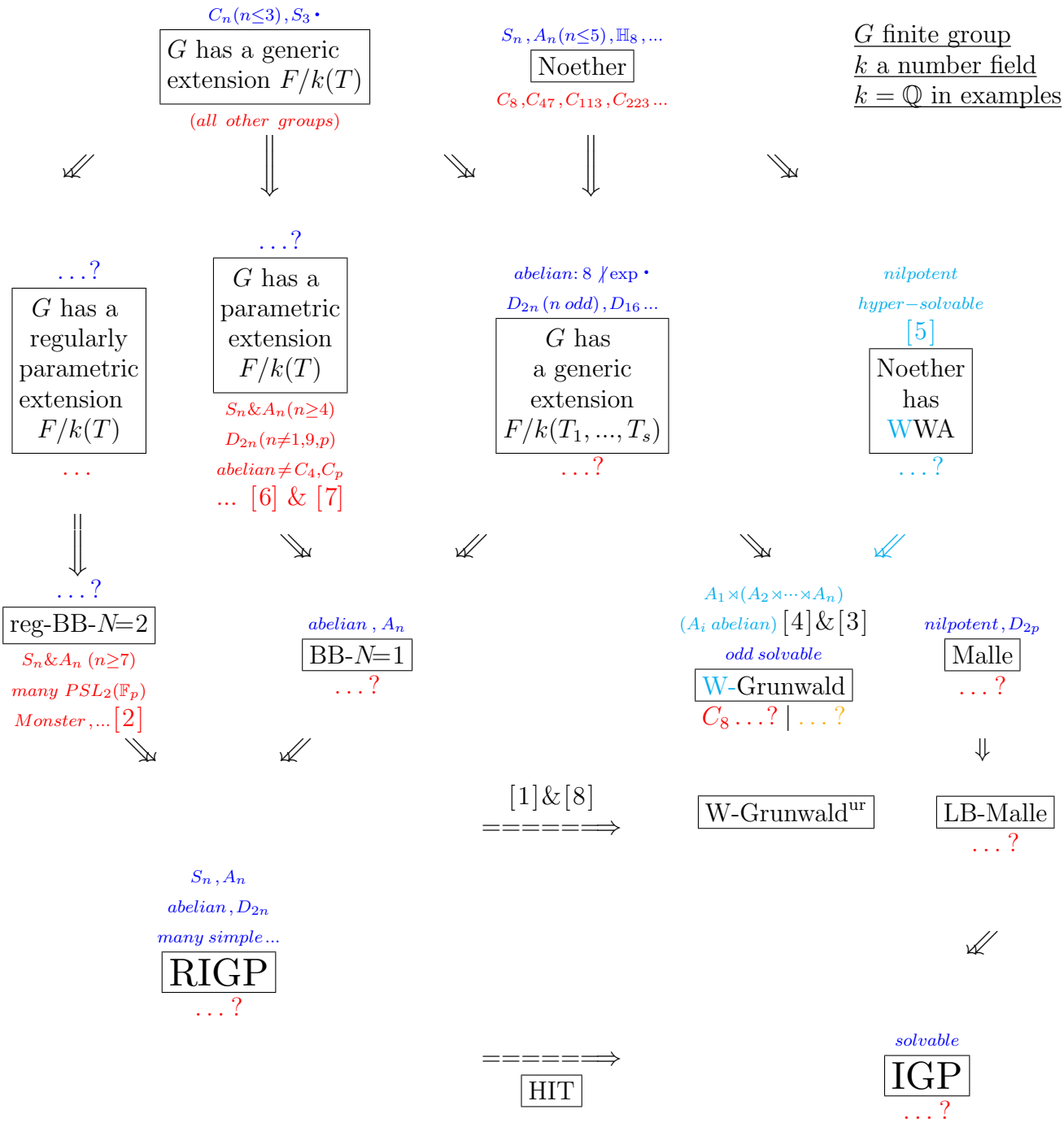
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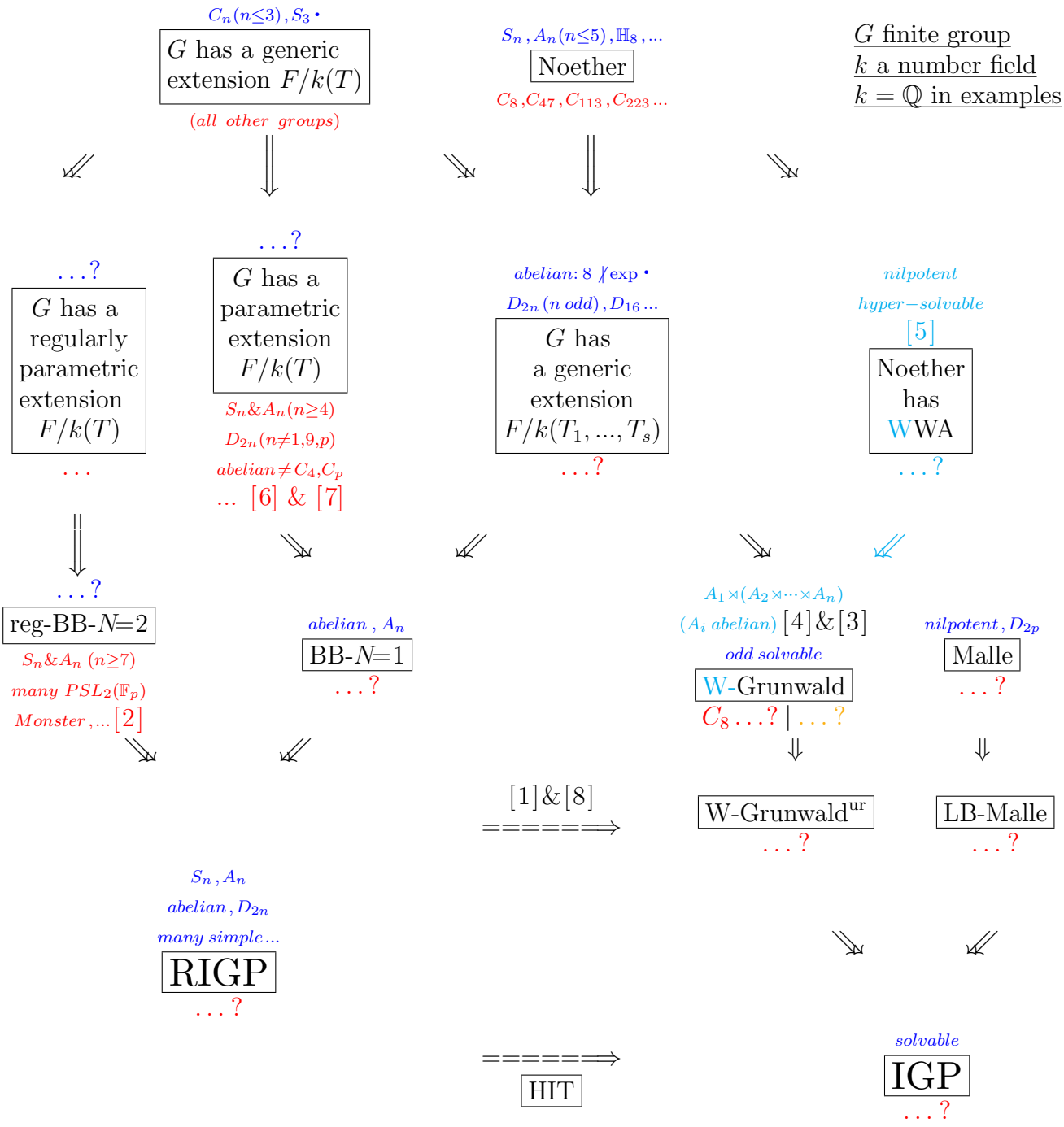
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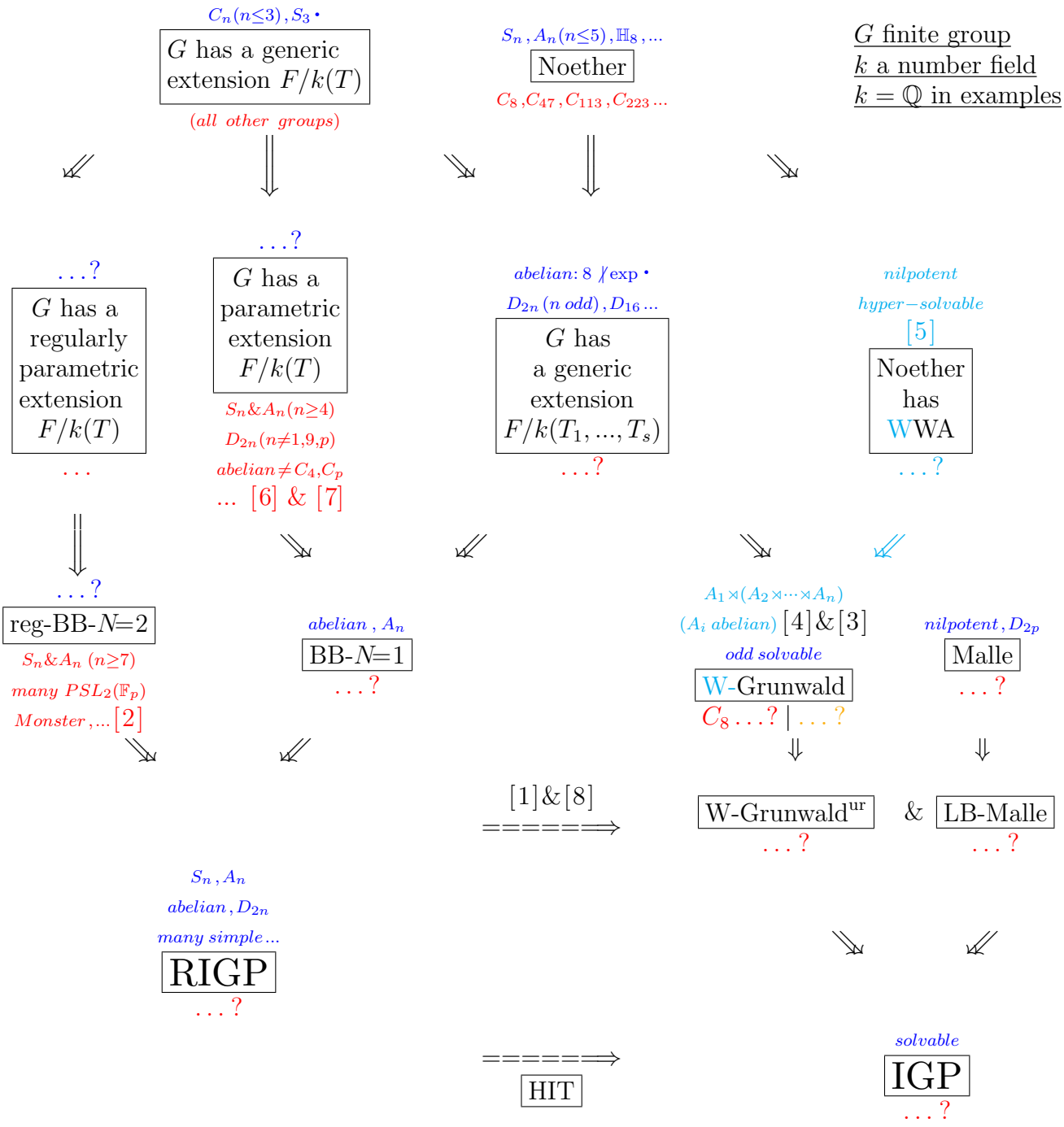
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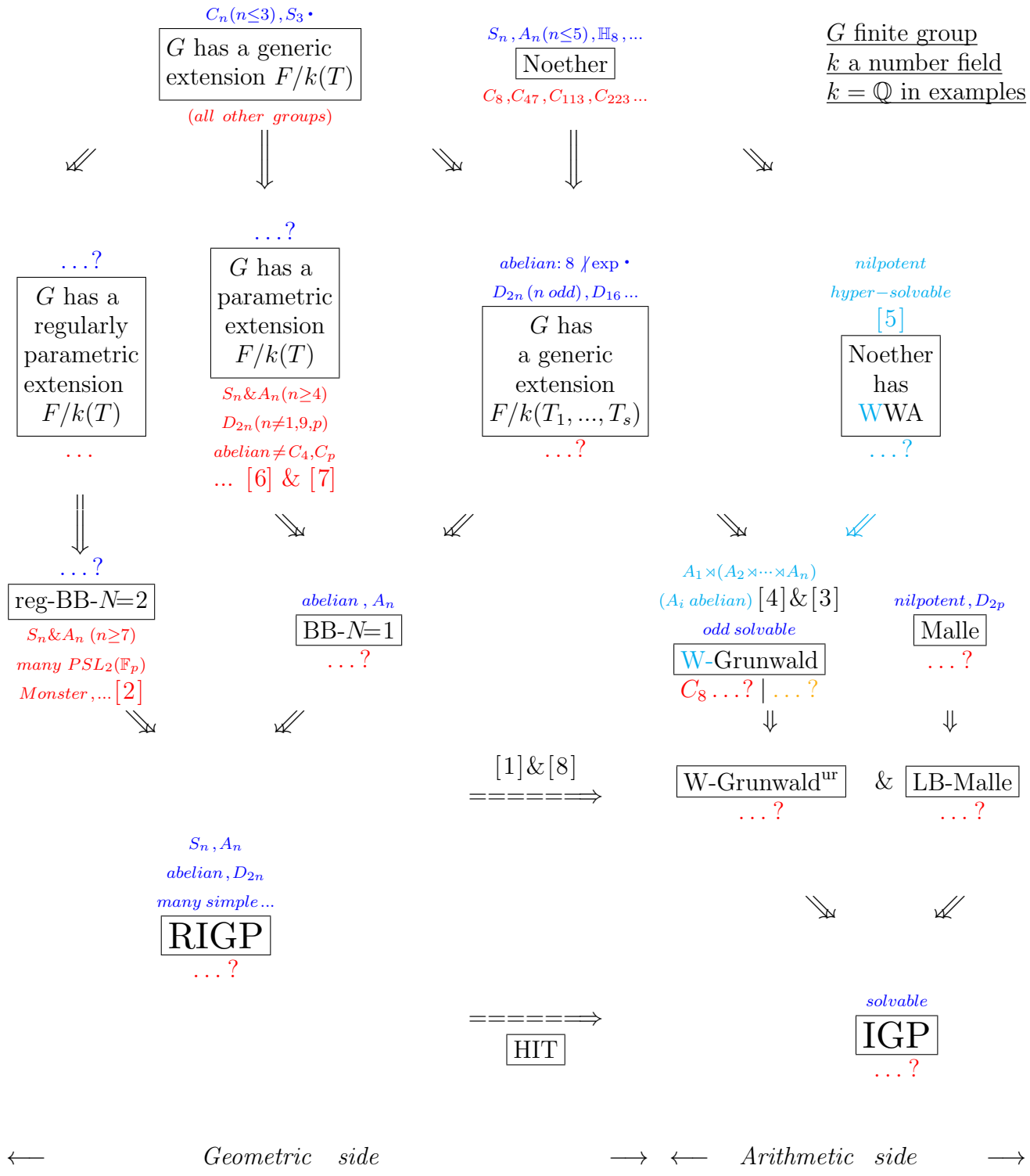
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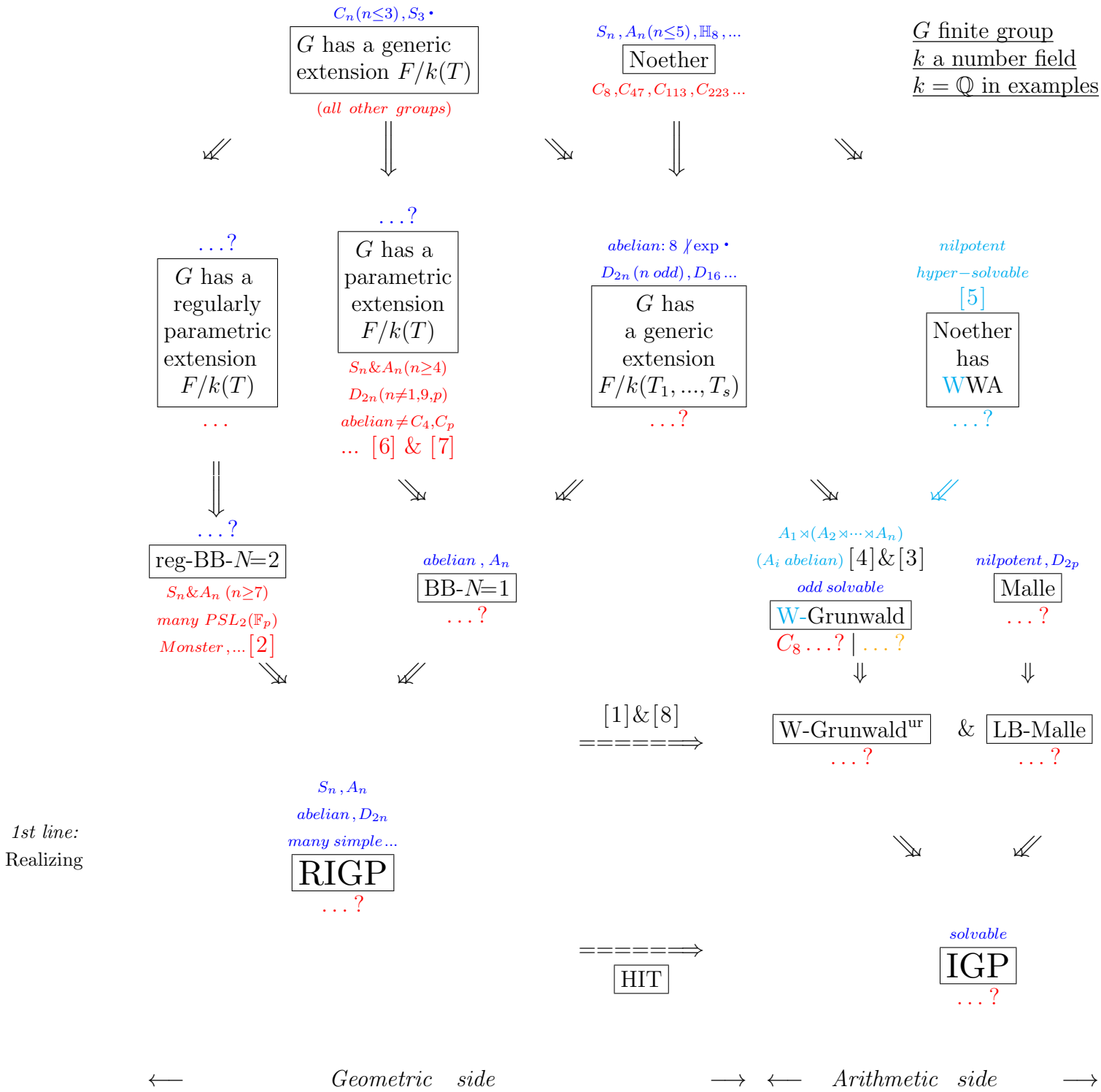
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1st line:
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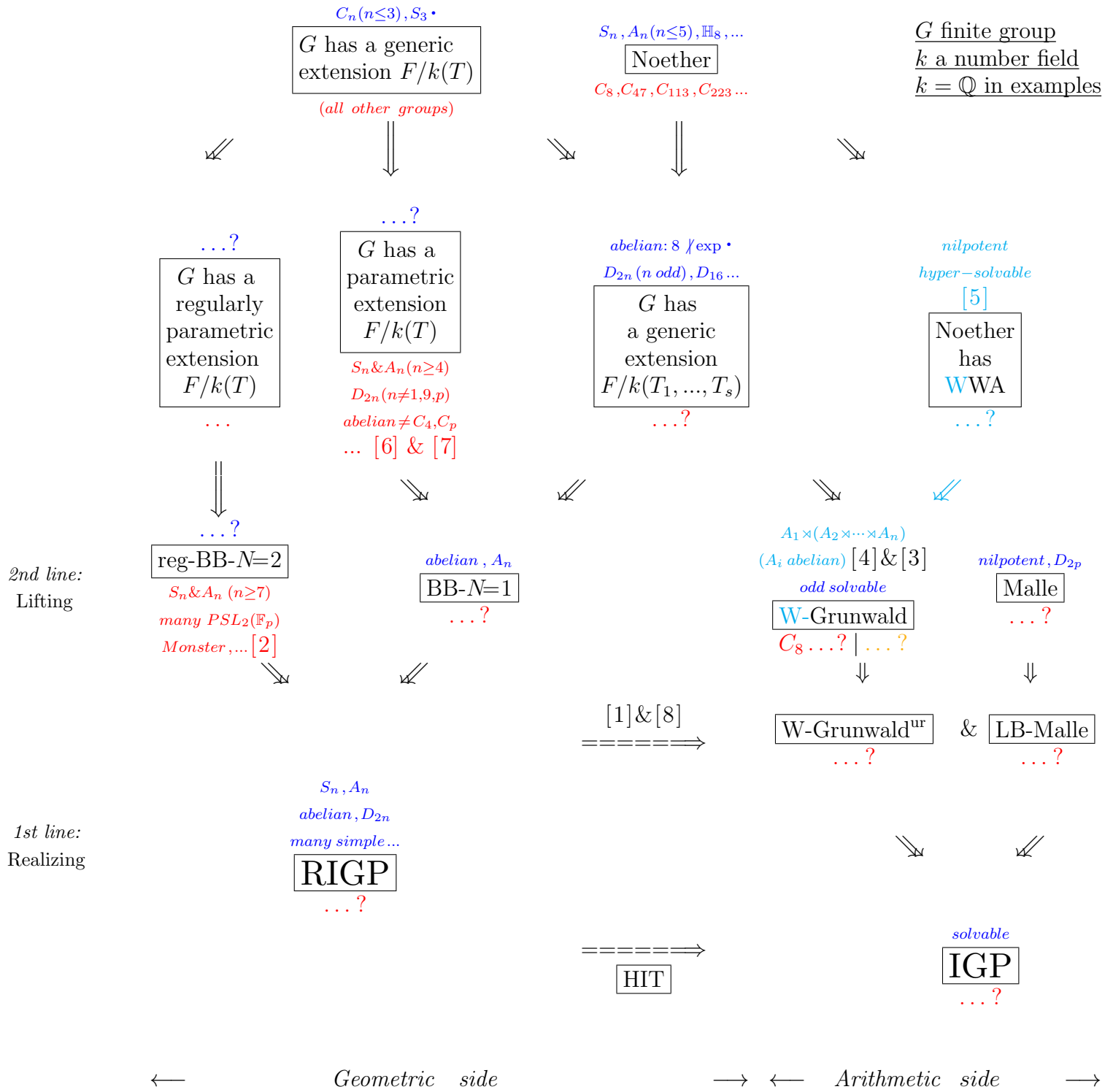
Geometric side
Arithmetic side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reg-BB-$N$=2
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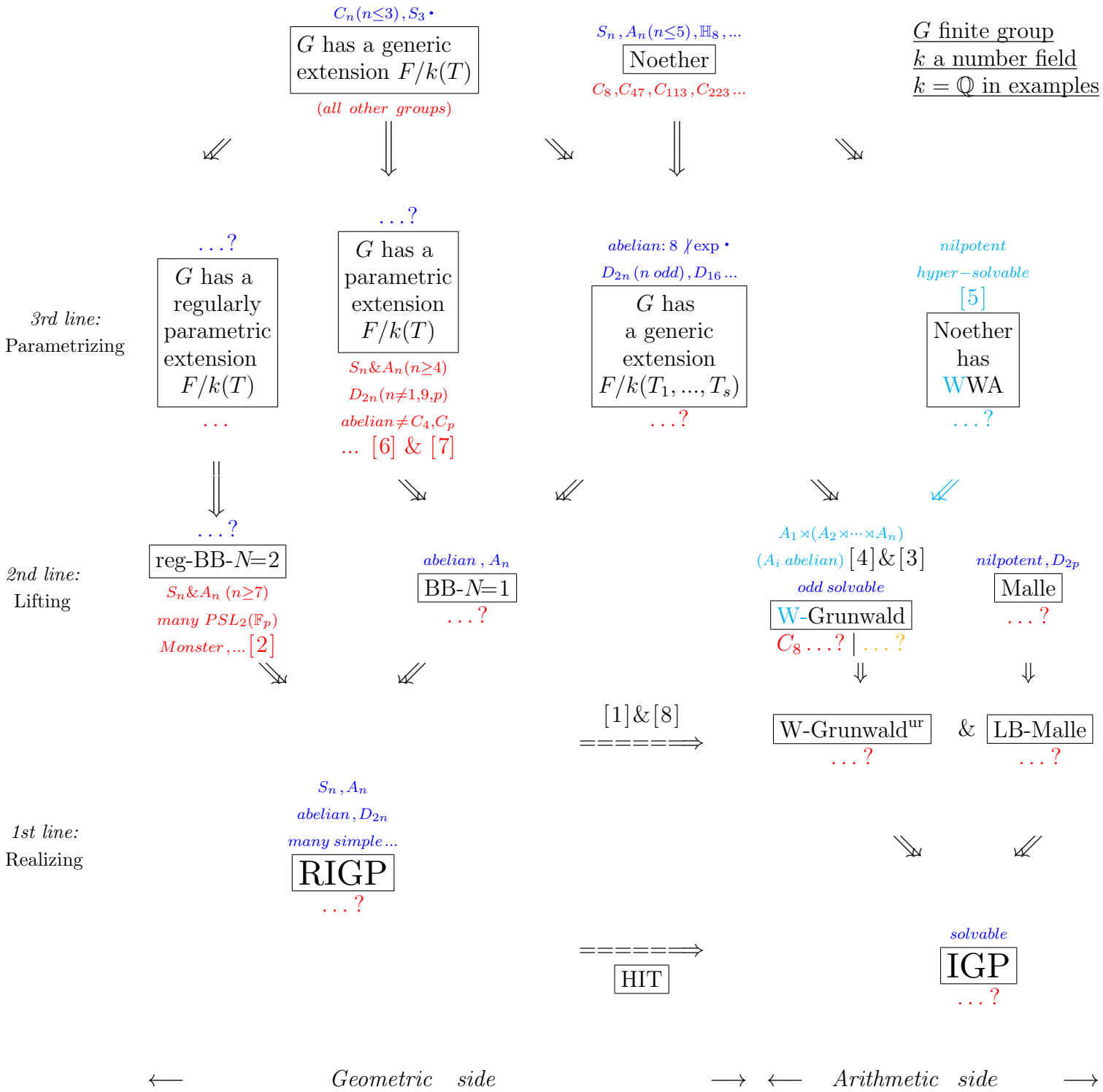
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3rd line:
Parametrizing
. . . ?
reg-BB-$N$=2
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2nd line:
Lifting
[1] & [8]
W-Grunwald$^{\mathrm{ur}}$
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1st line:
Realizing
solvable
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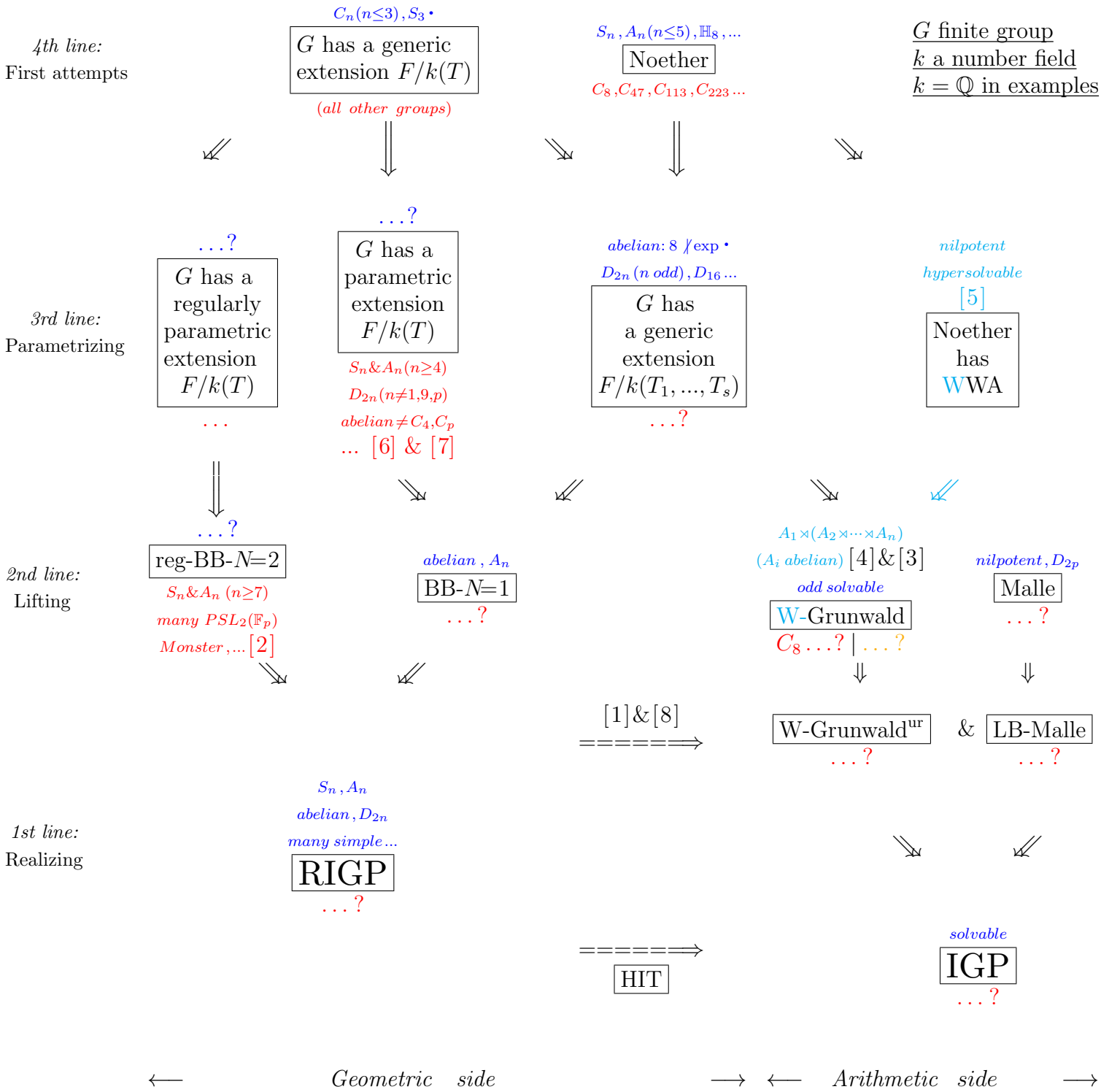
4th line:
First attempts
3rd line:
Parametrizing
2nd line:
Lifting
1st line:
Realizing
G finite group
k a number field
k = ℚ in examples
C_n (n≤3), S_3 •
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reg-BB-N=2
S_n & A_n (n≥7)
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