

Abstracts

- Francois Baccelli (INRIA ,ENS)
Information-Theoretic Capacity and Error Exponents of Stationary Point Processes under Random Additive Displacements
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Abstract : This paper studies the Shannon regime for the random displacement of stationary point processes. Let each point of some initial stationary point process in \mathbb{R}^n give rise to one daughter point, the location of which is obtained by adding a random vector to the coordinates of the mother point, with all displacement vectors independently and identically distributed for all points. The decoding problem is then the following one : the whole mother point process is known as well as the coordinates of some daughter point ; the displacements are only known through their law ; can one find the mother of this daughter point ? The Shannon regime is that where the dimension n tends to infinity and where the logarithm of the intensity of the point process is proportional to n . We show that this problem exhibits a sharp threshold : if the sum of the proportionality factor and of the differential entropy rate of the noise is positive, then the probability of finding the right mother point tends to 0 with n for all point processes and decoding strategies. If this sum is negative, there exist mother point processes, for instance Poisson, and decoding strategies, for instance maximum likelihood, for which the probability of finding the right mother tends to 1 with n . We then use large deviations theory to show that in the latter case, if the entropy spectrum of the noise satisfies a large deviation principle, then the error probability goes exponentially fast to 0 with an exponent that is given in closed form in terms of the rate function of the noise entropy spectrum. This is done for two classes of mother point processes : Poisson and Matérn. The practical interest to information theory comes from the explicit connection that we also establish between this problem and the estimation of error exponents in Shannon's additive noise channel with power constraints on the codewords.

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- Pierre Calka (Rouen)

Visibility estimates in Euclidean or hyperbolic germ-grain models and line tessellations

Abstract : The classical continuum percolation model (or germ-grain model) is considered in the Euclidean space and hyperbolic plane. When grains are homogeneously distributed, the critical intensity for the existence of an infinite connected component of the occupied (resp. unoccupied) phase has been studied in the Euclidean case notably by R. Meester and R. Roy and this work has been recently extended to the hyperbolic case by J. Tykesson. In this talk, we are interested in the visibility percolation which consists in asking whether it is possible to see to infinity in at least one direction. In the Euclidean case, we show that it is almost surely not possible to see to infinity for any dimension unless the intensity measure of the underlying point process is modified accordingly. In the hyperbolic space, I. Benjamini, J. Jonasson, O. Schramm & J. Tykesson have proved that there exists a critical intensity for the visibility percolation. We present precise estimates for the distribution tail of the maximal visibility in the subcritical and critical regimes. Moreover, we consider some generalizations of the model and we show asymptotic results when the intensity goes to zero or to the critical value. This is joint work with Johan Tykesson.

- Joël Chadoeuf (INRA)

Estimating the backward dispersion function of pollen trees

Joint work with F. Carpentier

Abstract : Efficient pollen dispersion, measured as the probability density that a pollen grain born at 0 falls at a given position, is classically estimated using genetic data collected on trees and on seeds attached to their mother trees. When all trees cannot be genotyped, classical methods based on maximum of likelihood cannot be used and Smouse and Austerlitz (2002) proposed to estimate a function, later called "backward dispersion function", using the probability for two seeds to have the same father, the backward dispersion function being the probability density that a seed on a tree at 0 has a father at a given position.

Current practices use indifferently this function or the classical definition, i.e. the probability density that a pollen grain

generated at 0 falls at a given position in space. Representation of biological processes as self-pollination are unclear in this modeling whereas the method depends heavily on the assumption that trees are Poisson distributed in space and on the assumption that genotypes are independently spread among trees.

We propose a critical analysis of the actual estimation method, show that this leads to biased estimators and show that the method imposes that no self-pollination exists. We then propose an estimation method which does not impose a Poisson assumption of tree spatial repartition in absence of self-pollination. Finally, we discuss how this last assumption or the assumption of random spread of genotypes among trees can be relaxed.

AUSTERLITZ F & SMOUSE PE. 2002. Two-Generation Analysis of Pollen Flow Across a Landscape. IV. Estimating the Dispersal Parameter. *Genetics*, Vol. 161, 355-363.

- Anne Estrade (Paris Descartes)
Covering the whole space with Poisson random balls
travail en collaboration avec Hermine Biermé (université Paris Descartes)

Abstract : We consider Poisson random balls, with the pair (center, radius) being given by a Poisson point process. According to the intensity measure of the Poisson process, we investigate the eventuality of covering the whole space with the union of the balls. We exhibit a disjunction phenomenon between the coverage with large balls (low frequency) and the coverage with small balls (high frequency). Concerning the second type of coverage, we prove the existence of a critical regime which separates the case where coverage occurs a.s. and the case where coverage does not occur a.s. We give an explicit value of the critical intensity and we prove that the Hausdorff measure of the set of points which are not covered by the union of balls is linked with this value. We also compare with other critical regimes appearing in continuum percolation.

- Jean-Baptiste Gouéré (Orléans)
Continuum percolation

Abstract : We consider the Poisson Boolean model of continuum percolation. At each point of an homogeneous Poisson point process of density λ , we center independent copies of a random ball. We then consider the union Σ of the random balls. We say that percolation occurs if one of the connected component of Σ is unbounded. One easily shows that percolation occurs if λ is large enough. In the main part of the talk, we will show that percolation does not occurs for small enough λ if and only if the mean volume of the random balls is finite. In the remaining part of the talk, we will consider related questions and related models.

- Kiên Kiêu & Clémence Kress (INRA, Jouy-en-Josas)
Towards a stochastic model of the spatial organization and the activity of mammalian genomes

Abstract : In eukaryotes, DNA is confined in the cell nucleus. Therefore it is in the nucleus that the main functions involving DNA such as replication, repair and transcription take place. DNA is highly folded in the nucleus : the whole genome extends on about 2 meters while the nucleus is about 5-10 microns. The wrapping of DNA around protein complexes (nucleosomes), forming the chromatin fiber, induces a first level of DNA compaction. However folding at a higher level is necessary to reach observed levels of compaction. Furthermore it is believed that the nucleus is not spatially homogeneous with respect to transcription : the densities of transcription factors and chromatin vary spatially. Conversely the activity of the genome may play a major role in the setup of its spatial organization. In particular, active loci tend to cluster together and could be responsible for establishing chromatin loops.

The interplay between the activity and the spatial organization of the genome has been investigated through diverse experimental methods. Modelling approaches have been developed to test hypotheses about rules governing the spatial organization. Models are generally stochastic and can be viewed as Gibbs models. Two types of geometry are considered : chains of fixed length segments (random walks) and sequences of points (worm-like chains). Different types of interactions are included : short-range repulsion (volume exclusion), attraction between consecutive points (compaction),

long-range attraction between a subset of points (looping). Such models succeed in reproducing global features of the chromatin spatial organization.

So far, models are low-dimensional and the chromatin fibers are supposed to be homogeneous. However recent high-throughput technics are providing data at the scale of the whole genome showing differences between various chromatin regions. Hence current models should be extended involving heterogeneous fibers instead of homogeneous ones. This means that interaction parameters should be specific to each segment or point, leading to models with thousands of parameters!

We will sketch such a model. The chromatin fibers will be represented as sequences of points (worm-like chains) where each point interacts specifically with other points and other nuclear components. In order to investigate the interplay between spatial organization and transcriptional activity, the transcription level of each point will be represented in the model as a mark. The different types of interactions to be considered will be modelled by potentials.

This model does not raise new theoretical issues concerning Gibbs model. However it is much more complex than standard models. This is due to the number of types of interactions considered in the model, and to the fact that points are allowed to interact specifically with other elements. Several issues relative to simulation and fitting will be discussed.

– C. Lantuéjoul (MinesParisTech)

The Poisson storm process

Extremal coefficients and simulation

This is joint work with J.N. Bacro (University Montpellier 2) and L.Bel (AgroParisTech).

Abstract : Introduced by Smith (1990) and extended by Schlather (2003), the storm process is a prototype of a max-stable random field. It has Fréchet margins for each compact support K , i.e. $P\{\max_{x \in K} Z(x) < z\} = \exp(-\theta(K)/z)$, which involves the *extremal coefficient* $\theta(K)$. Interpreting θ as a Choquet capacity makes it possible to derive a number of compatibility relationships linking the extremal coefficients for different supports. Here the particular case is considered where the storms are indicator functions of Poisson polygons

or polytopes. Explicit formulae are obtained for the extremal coefficients with finite or convex supports. Using importance sampling techniques, an algorithm is also proposed for simulating a two-dimensional Poisson storm process, without any approximation.

- Marie-Colette van Lieshout (CWI and Eindhoven University of Technology)

A J-function for inhomogeneous point processes with applications

Abstract : The analysis of data in the form of a map of (marked) points often starts with the computation of summary statistics. Some statistics are based on inter-point distances, others on the average number of points in sample regions, or on geometric information. For a survey of the state of the art and a rich source of pointers to the literature, the reader is referred to the recent Handbook of Spatial Statistics.

In the exploratory stage, it is usually assumed that the data constitute a realisation of a stationary point process and deviations from a homogeneous Poisson process are studied to suggest a suitable model. Although stationarity is a convenient assumption, especially if - as is often the case - only a single map is available, in many areas of application, though, heterogeneity IS present. To account for possible non-stationarity, Baddeley et al. (2000) defined a reduced second moment function by considering the random measure obtained from the mapped point pattern by weighting each observed point according to the (estimated) intensity at its location. Gabriel and Diggle (2009) took this idea further into the domain of space time point processes.

In this talk, we describe an extension of the J-function that is able to accommodate spatial and/or temporal inhomogeneity and, time-permitting, illustrate the approach to data on Pakistan earthquakes.

References : Baddeley, A.J., Moeller, J. and Waagepetersen, R. (2000). Non- and semi-parametric estimation of interaction in inhomogeneous point patterns, *Statistica Neerlandica*, 54, 329-350.

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- Ilya Molchanov (Bern)

Regularity conditions in the realisability problem in applications to point processes and random closed sets

Joint work with Raphael Lachieze-Rey (Lille and Luxembourg).

Abstract : The talk addresses the existence issue for a rather general random element whose distribution is only partially specified. The technique relies on the existence of a positive extension for linear functionals accompanied by additional conditions that ensure the regularity of the extension needed for interpreting it as a probability measure. It is shown in which case the extension can be chosen to possess some invariance properties.

The results are applied to obtain existence results for point processes with given correlation measure and random closed sets with given two-point covering function or contact distribution function. The regularity conditions ensure that the obtained point processes are indeed locally finite and random sets have closed realisations.

- Christoph Thäle (Osnabrück)

Iteration stable random tessellations via martingales

Abstract : Modern stochastic geometry has a growing demand for non-trivial and flexible, yet mathematically tractable models for spatial random tessellations. Iteration stable random tessellations form a relatively new class, have attracted quickly considerable interest and clearly show the potential to serve as new mathematical reference model for spatial crack structures. In my talk I will briefly introduce iteration stable random tessellation and show how methods from martingale theory may successfully be applied to give new insight to the first- and second-order structure of these tessellations. In particular, combined with tools from integral geometry, these methods allow the exact calculation of certain variances and covariances, which were out of reach before. Moreover, martingale limit theorems are used to conclude limit theorems for the random tessellations under consideration. (This

is joint work with Tomasz Schreiber, who died untimely in December 2010 at the age of 35.)

- Hans Zessin (Bielefeld)

On Polya sum and difference point processes

Abstract : Motivated by the quantum mechanical Bose and Fermi gas I shall construct in an elementary context two new point processes, which we call Polya sum resp. Polya difference process, and discuss their properties. They seem to be as fundamental as the Poisson point process.

- Sergei Zuyev (Chalmers University of Technology)

Sequential adsorbing point process

Abstract : We study a class of dynamically constructed point processes in which at every step a new point is added to the current configuration with a distribution depending on the local structure around a uniformly chosen point. This class covers, in particular, generalised Polya urn scheme, Dubbins-Freedman random measures and cooperative sequential adsorption models studied previously. We address boundedness of the processes and convergence properties of the corresponding sample measure. We show that in general the limiting measure is random when exists and that this is the case for a wide class of almost surely bounded processes.