POISSON CYLINDERS IN EUCLIDEAN AND HYPERBOLIC GEOMETRY

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We consider a homogeneous Poisson line process in Euclidean space $\mathbb{R}^d$ or hyperbolic space $\mathbb{H}^d$ (for $d \geq 2$) of intensity $u \in (0, \infty)$. Around each line in the Poisson line process a solid bi-infinite cylinder of base-radius 1 is centered. This continuum percolation model is called the Poisson cylinder model. Let $\mathcal{C}$ denote the union of all the cylinders in the process and let $\mathcal{V}$ denote the complement of $\mathcal{C}$. In $\mathbb{R}^d$ we show that conditioned on the event that the two points x and y belong to the set $\mathcal{C}$, there is a.s. a sequence of d cylinders $\mathfrak{c}_1, \ldots, \mathfrak{c}_d$ from the process such that

$$x, y \in \cup_{i=1}^d \mathfrak{c}_i \quad \text{and} \quad \cup_{i=1}^d \mathfrak{c}_i \quad \text{is connected.}$$

In Hyperbolic space $\mathbb{H}^d$, we show that there is a critical intensity $u^*(d) \in (0, \infty)$ such that $\mathcal{C}$ is a.s. disconnected for $u < u^*$, and a.s. connected for $u > u^*$. Some natural open problems for the Poisson cylinder model will also be discussed. The talk is based on joint works with Erik Broman from Uppsala University.