The Unruh effect revisited

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INTRODUCTION

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"How does a vacuum field look to an accelerated (rather than inertial) observer? Or, similarly, what happens if a detector is accelerated through a vacuum field?"

Answer: When a detector, coupled to a relativistic quantum field in its vacuum state, is uniformly accelerated through Minkowski spacetime, with proper acceleration a, it registers a **thermal** black body radiation at temperature $T = \frac{\hbar a}{2\pi c k_{\mathrm{B}}} \sim 10^{-19}a$. In other words, it perceives a thermal bath of **particles**.

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SURPRISED?

- If you think the vacuum is empty space, you will/should be: how can a detector, accelerated or not, see **particles** if there aren't any?

- If you remember the vacuum is the ground state of the field, you should NOT be surprised the detector reacts to the presence of the field. What is STILL surprising is the claim that it "perceives" a **thermal** distribution of radiation. Indeed, what this means precisely is that, tracing out the field variables in the coupled field-detector system, the state of the detector converges, asymptotically in time, to the Gibbs state at temperature T: after all, a priori, it could have been any other mixed state.

GOAL Explain this last claim, for the standard model of the detector-field system.

PLAN: Recipe for an Unruh effect without particles

INGREDIENTS:

- A whiff of special relativity:uniform acceleration and the Rindler wedge

- A sprinkling of free quantum field theory: the Klein-Gordon field

- One detector

Prepare both detector and field in their ground state. Throw the detector in with the field as in a spin-boson model. Accelerate the detector. Wait. Recover the detector when it reaches the desired temperature.

A whiff of special relativity

Uniform acceleration The worldline $x^0(\sigma), \mathbf{x}(\sigma)$, parametrized by its proper time $\sigma \in \mathbb{R}$, describes the trajectory of a uniformly accelerated observer (or particle, or detector) with proper acceleration $a > 0$, provided

$$(\ddot{x}^0(\sigma))^2 - (\ddot{\mathbf{x}}(\sigma))^2 = a^2, \quad (\dot{x}^0(\sigma))^2 - (\dot{\mathbf{x}}(\sigma))^2 = 1.$$

This implies that, in an adapted choice of inertial coordinate frame,

$$x^0(\sigma) = \frac{1}{a}\sinh a\sigma, \quad x^1(\sigma) = \frac{1}{a}\cosh a\sigma, \quad x^2(\sigma) = 0 = x^3(\sigma).$$
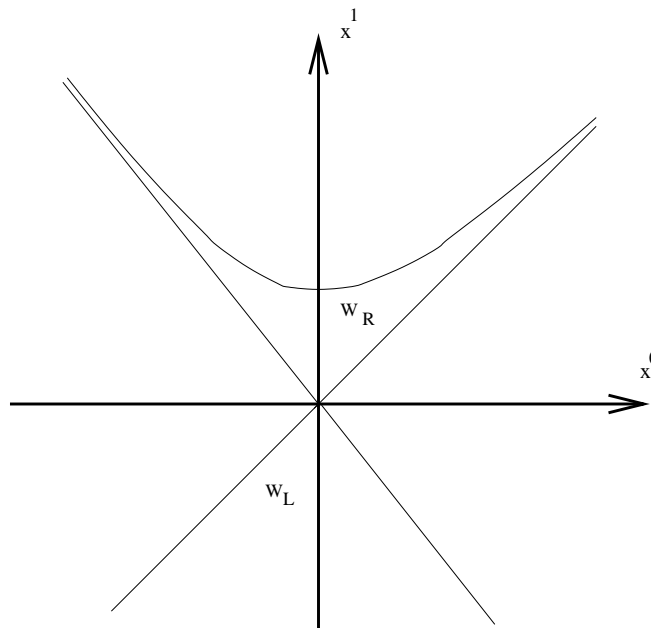
In the laboratory frame, the particle's speed tends to the speed of light in the distant past and future.

The Rindler wedge Associated to this worldline is the wedge W_{R} where $x^1 > |x^0|$ and where there exist global co-moving coordinates: $(\tau, u, \underline{x}_\perp)$, defined as follows (Rindler coordinates, hyperbolic coordinates):

$$x^0 = u \sinh \tau, \ x^1 = u \cosh \tau, \underline{x}_\perp = (x_2, x_3).$$

$u > 0$ is a spatial coordinate, τ is a time coordinate.

Note that $W_{\mathrm{L}} := -W_{\mathrm{R}}$ is the causal complement of $\overline{W}_{\mathrm{R}}$.

Each curve $u = 1/a'$, $a' > 0$, is the worldline of a uniformly accelerating particle, with proper acceleration a'. Its proper time is $\sigma = \tau/a'$. Two such particles remain equidistant at all times, as seen from either of their instantaneous restframes.

Below, we will consider a detector moving with acceleration $a = 1$.

In Rindler coordinates, the Minkowski line element becomes:

$$\mathrm{d}s^2 = u^2\mathrm{d}\tau^2 - \mathrm{d}u^2 - \mathrm{d}\underline{x}_\perp^2.$$

And the Klein-Gordon equation $\left(-\partial_{x^0}^2 + \Delta + m^2\right)q(x^0, \mathbf{x}) = 0$ reads

$$\left(u^{-2}\partial_\tau^2 - u^{-1}\partial_u u \partial_u + (-\Delta_\perp + m^2)\right) q(\tau, u, \underline{x}_\perp) = 0.$$

A sprinkling of free quantum field theory: the Klein-Gordon field

The radiation field differs from atomic systems principally by

... having an infinite number of degrees of freedom. This may

cause some difficulties in visualizing the physical problem,

but is not, in itself, a difficulty of the formalism.

R. Peierls

The field A Klein-Gordon field $Q(x)$ on Minkowski spacetime *is*, by definition, a family of self-adjoint operators $Q(x)$, labeled by spacetime points $x = (x^0, \underline{x})$ satisfying

$$\partial_{x^0}^2 Q(x) = -(-\Delta + m^2)Q(x), \quad [Q(x^0, \underline{x}), \partial_{x^0} Q(x^0, \underline{x}')] = \mathrm{i}\delta(\underline{x} - \underline{x}').$$

or, writing $t = x^0$ and $\Omega^2 = -\Delta + m^2$,

$$\partial_t^2 Q(t, \underline{x}) = -\Omega^2 Q(t, \underline{x}) \quad \text{i.e.} \quad \partial_t^2 Q(t) = -\Omega^2 Q(t)$$

QUESTION : Why is that? And how to construct such an object?

Hint: Believe Peierls.

ANSWER : (a) View $Q(t, \underline{x})$ as the displacement of a harmonic oscillator degree of freedom at the spatial point $\underline{x}$ and at time x^0 : HEISENBERG PICTURE!! (b) Notice the analogy with a finite family of oscillators $Q = (Q(1), Q(2), \ldots, Q(N))$, where the discrete and finite index i replaces $\underline{x} \in \mathbb{R}^3$, with Hamiltonian

$$H = \frac{1}{2}P^2 + \frac{1}{2}Q \cdot \Omega^2 Q, \quad \Omega^2 \text{ a positive } n \times n \text{ matrix.}$$

Then $Q(t, j) := \mathrm{e}^{itH} Q(j)\mathrm{e}^{-itH}$ satisfies the Heisenberg equations of motion

$$\partial_t^2 Q(t, j) = -\Omega_{jk}^2 Q(t, k), \quad \text{i.e.} \quad \partial_t^2 Q(t) = -\Omega^2 Q(t).$$

and the equal time commutation relations $[Q(t, j), \partial_t Q(t, k)] = \mathrm{i}\delta_{jk}$. (c) Recall

$$Q(t, j) = \sum_{\ell=1}^{N} \frac{1}{\sqrt{2\omega_\ell}} \left(\eta_\ell(j) \, e^{-\mathrm{i}\omega_\ell t} a_\ell + \overline{\eta}_\ell(j) \, e^{\mathrm{i}\omega_\ell t} a_\ell^\dagger \right) \quad \Omega^2 \eta_\ell = \omega_\ell^2 \eta_\ell.$$

(d) Write the analogous expression for the Klein-Gordon field and check it works:

$$Q(t, \underline{x}) = \int_{\mathbb{R}^3} \frac{\mathrm{d}\underline{k}}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\omega(\underline{k})}} (e^{\mathrm{i}\underline{k} \cdot \underline{x}} \, e^{-\mathrm{i}\omega(\underline{k})t} \, a(\underline{k}) + e^{-\mathrm{i}\underline{k} \cdot \underline{x}} \, e^{\mathrm{i}\omega(\underline{k})t} \, a^\dagger(\underline{k})).$$

CONCLUSION : This will do the job! So, the quantum Klein-Gordon field is "just" an infinite dimensional version of the harmonic oscillator. (Thank you Mr. Peierls!)

OBSERVABLES : For a system of harmonic oscillators, the observables are "all functions of $Q(j)$ and $P(j) = \dot{Q}(j)$", or, equivalently, all functions of $Q(j,t), j = 1 \ldots N, t \in \mathbb{R}$. Similarly, for the Klein-Gordon fields, the observables are all functions of $Q(x^0, \underline{x})$, with $(x^0, \underline{x})$ running through space-time.

THE VACUUM $|0\rangle$: As for a system of harmonic oscillators, it is the state of lowest energy. It is determined by its two-point function :

$$\langle 0|Q(0,\underline{x})Q(0,\underline{y})|0\rangle = (-\Delta + m^2)^{-1/2}(\underline{x},\underline{y}), \cdots .$$

Compare : $\langle 0|Q(i,0)Q(j,0)|0\rangle = \Omega_{ij}^{-1/2}$ for a system of n oscillators satisfying $\ddot{Q}(i,t) = -\Omega_{ij}^2 Q(j,t)$.

Notice: There is no need to talk about particles anywhere. Quantum field theory is about fields, not particles.

Throw in a detector

The detector is modeled by a two-level system. And "detection of a particle", for those who like to think in those terms, is defined as "excitation of the detector." Its observable algebra is generated by fermionic creation/annihilation operators $A(\tau)$, $A^\dagger(\tau)$ satisfying

$$A(\tau)A^\dagger(\tau) + A^\dagger(\tau)A(\tau) = \mathbb{1}, \qquad A^2(\tau) = 0.$$

The free Heisenberg evolution of the detector is:

$$\dot{A}(\tau) = -iEA(\tau) = i[H_{\mathrm{D}}, A(\tau)] \qquad \text{where } H_{\mathrm{D}} = EA^\dagger A$$

and where τ is the detector's proper time and $A := A(0)$.

We will start the detector of in its ground state defined by

$$\langle A \rangle_{\mathrm{g,D}} = 0, \qquad \langle H_{\mathrm{D}} \rangle_{\mathrm{g,D}} = 0.$$

We will show it ends up in a thermal state at inverse temperature $\beta > 0$, defined by

$$\langle A \rangle_{\beta,\mathrm{D}} = 0, \qquad \langle A^\dagger A \rangle_{\beta,\mathrm{D}} = \frac{\mathrm{e}^{-\beta E}}{1 + \mathrm{e}^{-\beta E}}.$$

Think of it as a spin-boson system, and remember Peierls. So, a natural way to couple a two-level system to an oscillator chain is this:

$$H = \frac{1}{2}(P^2 + Q \cdot \Omega^2 Q) + E A^\dagger A + \lambda(A + A^\dagger)Q(0).$$

This corresponds to the spin $1/2$ being coupled to the oscillator at the origin $j = 0$. The Heisenberg equations of motion of this system are:

$$
\begin{aligned}
\partial_t^2 Q(t,j) &= -\Omega_{jk}^2 Q(t,k) - \lambda(A + A^\dagger)(t)\delta_{j0} \\
\dot{A}(t) &= -\mathrm{i}E A(t) + \mathrm{i}\lambda[A(t), A^\dagger(t)]Q(0,t)
\end{aligned}
$$

Note that in this picture we think of the two-level system as sitting still at the origin of space.

QUESTION: If we put at $t = 0$ the system in its unperturbed ground state, and imagine the oscillator chain is infinite, do we expect the state of the two-level system to converge after a long time to a thermal state at some positive temperature?

Whipping up the Unruh effect

The coupled Heisenberg equations of motion For $\rho \in C_0^\infty$

$$(\Box + m^2)Q(\tau, u, \underline{x}_\perp) = \lambda \rho(\| x_* \|)(A + A^\dagger)(\tau)$$

$$x \in W_R, x_* := (0, u - 1, \underline{x}_\perp)$$

$$(\Box + m^2)Q(x) = 0, \quad \text{otherwise}$$

$$\dot{A}(\tau) = -\mathrm{i}EA(\tau) + \mathrm{i}\lambda C(\tau) \int \mathrm{d}u \mathrm{d}\underline{x}_\perp \rho(\| x_* \|)Q(\tau, u, \underline{x}_\perp),$$

with $C(\tau) = [A(\tau), A^\dagger(\tau)]\theta(\tau)$.

The Unruh effect (b) $\displaystyle\lim_{\tau \to \infty} \langle (A^\dagger A)(\tau) \rangle_g = \frac{\mathrm{e}^{-\beta E}}{1 + \mathrm{e}^{-\beta E}} + \mathrm{O}(\lambda^2), \beta = 2\pi.$

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Elements of an explanation

I. GROUND STATES CAN LOOK THERMAL Restricted to the Rindler wedge, the Minkowski ground state has thermal properties, with respect to the Rindler time coordinate. In particular, it is a mixed state!

II. CAUSALITY The field variables $Q(x^0, \underline{x})$ with $(x^0, \underline{x})$ inside the right Rindler wedge (where the detector is) are causally independent of the $Q(0, \underline{x})$, with $x^1 < 0$.

These two facts imply that, from the detector's point of view, everything happens as if it is coupled to an infinite thermal reservoir. Hence its state will evolve to the thermal equilibrium at that temperature.

Let us comment on both these points, and in particular explore where the analogy with an ordinary non-relativistic spin-boson model breaks down.

A simple example Consider two coupled oscillators ($0 \leq \gamma < 1$)

$$H = \frac{1}{2}P^2 + Q \cdot \Omega^2 Q, \quad \Omega = \omega \begin{pmatrix} 1 & \gamma \\ \gamma & 1 \end{pmatrix}.$$

Consider also the Hamiltonian

$$H_* = \frac{1}{2}P_1^2 + \frac{1}{2}\omega_*^2 Q_1^2 + \frac{1}{2}P_2^2 + \frac{1}{2}\omega_*^2 Q_2^2 = H_{*1} + H_{*2},$$

where $\omega_* = \sqrt{1-\gamma}\,\omega$. Then, with $\beta = \frac{1}{\omega_*} \ln \frac{1+\sqrt{1-\gamma}}{1-\sqrt{1-\gamma}}$,

$$\langle 0|A(Q_1, P_1)|0\rangle = \frac{1}{Z_{*1}} \mathrm{Tr}\, \mathrm{e}^{-\beta H_{*1}} A(Q_1, P_1).$$

The ground state of H, which is a pure state of the total system, becomes a mixed state when restricted to a subsystem. This is no surprise; what is remarkable is that this mixed state is the thermal state of another harmonic oscillator, for some specific positive temperature. Something similar lies at the root of the Unruh effect.

Thermality of the KG field on the Rindler wedge

THEOREM (Bisognano-Wichmann 75 – Sewell 82) The Minkowski vacuum has the following property, referred to as the KMS condition

$$\langle 0|Q(0, u, \underline{x}_\perp)Q(\tau + \mathrm{i}2\pi, u', \underline{x}'|0\rangle = \langle 0|Q(\tau, u', \underline{x}'_\perp)Q(0, u, \underline{x}|0\rangle$$

Compare this to

$$\frac{1}{Z_\beta}\mathrm{Tr}\,e^{-\beta H}AB(t + i\beta) = \frac{1}{Z_\beta}\mathrm{Tr}\,e^{-\beta H}B(t)A,$$

which is known to characterize thermal equilibrium in systems with a finite number of degrees of freedom. So,

"If it looks like a duck, walks like a duck, talks like a duck..."

Note that in the case of the KG-field on the Rindler wedge, it is with respect to the Rindler time, which is the detector's proper time, not the laboratory time, that the state is thermal (of course!!).

II. The role of causality

Looking again at the Heisenberg equations of motion you see the field operators $Q(x^0, \underline{x})$ for x in the Right wedge, where the detector finds itself, are, due to causality, dynamically decoupled from the ones in the left wedge and in particular, that no knowledge about the $Q(0, x^1, \underline{x}_\perp)$ for $x_1 < 0$ is required to determine the $Q(x)$ with x in the right wedge. The $Q(x)$ with x in the right wedge form, together with the detector observables an autonomous system.

Let's illustrate this with an analogous (but rather artificial) phenomenon in the simple example of the two coupled oscillators introduced above.

A simple example (bis) Recall ($0 \leq \gamma < 1$)

$$H = \frac{1}{2}P^2 + Q \cdot \Omega^2 Q, \quad \Omega = \omega \begin{pmatrix} 1 & \gamma \\ \gamma & 1 \end{pmatrix}.$$

Then, with $\omega_* = \sqrt{1-\gamma}\omega$, $\beta = \frac{1}{\omega_*} \ln \frac{1+\sqrt{1-\gamma}}{1-\sqrt{1-\gamma}}$,

$$\langle 0|A(Q_1, P_1)|0\rangle = \frac{1}{Z_{*1}} \mathrm{Tr}\, e^{-\beta H_{*1}} A(Q_1, P_1), \quad H_{*1} = \frac{1}{2}P_1^2 + \frac{1}{2}\omega_*^2 Q_1^2$$

Suppose now you let, for $t > 0$, the system evolve with the Hamiltonian

$$\tilde{H} = H_{*1} + H_{*2} + \lambda(A + A^\dagger)Q_1.$$

This, you can (almost) achieve by snapping the connection between the two oscillators. Then the two-level system will feel "bathed in a thermal bath of quanta" of the Hamiltonian H_{*1} and react accordingly. The snapping of the connection is artificial here, but provided in the KG equation by causality!

Particles? What particles?

CLAIM: "…for a free quantum field in its vacuum state in Minkowski spacetime an observer with uniform acceleration a feels he is bathed by a thermal distribution of field quanta at temperature $T = \hbar a / 2\pi c k_B$." (Unruh and Wald, 1984)

EXPLANATION: the quanta are Rindler quanta. Fulling proved (1973) that the notion of vacuum and hence of particle or better, quantum, is observer-dependent and notably that on the Rindler wedge the vacuum with respect to the Rindler time is not identical to the Minkowski vacuum, associated with inertial observers.

In fact, the Rindler creation/annihilation operators are related to the Minkowski ones via a Bogoliubov transformation. The Minkowski Fock space can (roughly) be written

$$\mathcal{F}_M = \mathcal{F}_{LW} \otimes \mathcal{F}_{RW},$$

and

$$| \, 0, M \rangle \sim \prod_i \sum_{n_i} \mathrm{e}^{-\beta \omega_i n_i} \frac{1}{n_i!} \left(a^\dagger_{LW,i} a^\dagger_{RW,i} \right)^{n_i} |0, \mathrm{Ri}\rangle.$$

This is Bisognano-Wichman in a different guise.

Tracing out the Left Wedge variables, you end up with a density matrix describing a thermal bath of Rindler particles on the right wedge. If you now couple a detector and compute perturbatively its excitation rate, you will find a result compatible with it being "bathed" in a thermal distribution of Rindler quanta (Unruh-Wald 1984).

QUESTION: Still, that sounds like fancy footwork. Does this not violate basic energy conservation? Since the field is in its (Minkowski) vacuum state, how can it excite the detector? And how can the excitation of the detector be accompanied by the emission of a Minkowski particle, since the field is in its vacuum state?

ANSWER: Because the detector is accelerating, energy is put into the system that can be transfered to the field. So simultaneous excitation of the detector AND of the field are perfectly natural. In fact, while the inertial laboratory observer will claim the detector has been excited, and simultaneously a field quantum has been emitted, an observer using Rindler coordinates, will agree the detector has been excited but will also say that a field quantum has been absorbed. The apparent contradiction stems from the fact they don't use the same meaning for the word "quantum".

A very personal conclusion

It seems exaggerated to identify the excitation of a two-level system with the "detection of a particle." To the least, it is too colourful a language. And to the extent that the particle concept is known to pose conceptual problems, it seems a good thing to understand the Unruh effect without referring to particles.

It turns out much of the mystery and controversy surrounding discussions of the Unruh effect goes away if you think in purely statistical mechanics terms. But the beauty remains, it seems to me. In a nutshell: due to causality, the two-level system is coupled only to a proper subset of all degrees of freedom of the field. The vacuum state of the field restricted to this subset has thermal properties. And since a small system coupled to an infinite reservoir at thermal equilibrium, evolves to that equilibrium, the Unruh effect emerges. .