Error Bounds and Estimates for Matrix Functions

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Let A be a square matrix, b a vector, and f a function which is analytic in a set Ω in the complex plane that contains the spectrum of A. For many such functions, $f(A)$ or $f(A)b$ are computed by first approximating f by a polynomial or rational function, and then evaluating the latter. These lectures will discuss approaches to bound or estimate the error in the computed approximations. We will first review error bounds for polynomial and rational approximation of analytic functions on regions in the complex plane and then discuss how these bounds shed light on convergence of the error for best polynomials and rational approximants as their degree or order is increased. Standard and rational Krylov subspace methods for the approximation of $f(A)b$ receive special attention. We define the Faber transform and show how it allows us to map approximation problems on simply connected sets in the complex plane to approximation problems on the unit disk. This transform also is helpful for computing near-best polynomial and rational approximants. Finally, we discuss results by Crouzeix on functions of matrices and operators. The lectures are based on the following reference:

References

[1] B. Beckermann, Image numérique, GMRES et polynômes de Faber, C. R. Acad. Sci. Paris, Ser. I, 340 (2005), pp. 855–860.

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[4] D. Gaier, Lectures on Complex Approximation, Birkhäuser, Boston, 1987: Chapter 1, §1-4, §6.

[5] J. L. Walsh, Interpolation and Approximation by Rational Functions in the Complex Domain, Amer. Math. Society, Providence, 1960: Chapter 4.