Sparse Interpolation

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A mathematical model is called t-sparse if it is a combination of only t generating elements. In sparse interpolation, the aim is to determine both the support of the sparse linear combination and the scalar coefficients in the representation, from a small or minimal amount of data samples. Sparse techniques solve the problem statement from a number of samples proportional to the number t of terms in the representation rather than the number of available data points or available generating elements. Sparse representations reduce the complexity in several ways: data collection, algorithmic complexity, model complexity.

We indicate the connections between sparse interpolation, coding theory, generalized eigenvalue computation, exponential analysis and rational approximation. In the past few years, insight gained from the computer algebra community combined with methods developed by the numerical analysis community, has lead to significant progress in several very practical and real-life signal processing applications. We make use of tools such as the singular value decomposition and various convergence results for Padé approximants to regularize an otherwise inverse problem. Classical resolution limitations in signal processing with respect to frequency and decay rates, are overcome.

In the illustrations we particularly focus on multi-exponential models

$$\phi(t) = \sum_{i=1}^{t} \alpha_i \exp(\phi_i t), \qquad \alpha_i = \beta_i + \mathtt{i}\gamma_i, \quad \phi_i = \psi_i + \mathtt{i}\omega_i, \tag{1}$$

representing signals which fall exponentially with time. These models appear, for instance, in transient detection, motor fault diagnosis, electrophysiology, magnetic resonance and infrared spectroscopy, vibration analysis, seismic data analysis, music signal processing, dynamic spectrum management such as in cognitive radio, nuclear science, and so on.