PROGRAM

Time	Thursday 28 June
9h 30 – 10h 00	Welcome of participants
10h 00– 10h 50	Norman Levenberg (Indiana University, Bloomington) Large deviation in certain vector energy settings
11h00 – 11h50	Alexander Borichev (Lab. LATP, Univ. Aix-Marseille) Two completeness problems in the Paley-Wiener space
12h 00 - 13h 45	Lunch (Restaurant Universitaire Charles Barrois)
14h 00 – 14h 50	Nick Trefethen (Oxford University, England) Highlights of ATAP
15h00 - 15h 50	Marc Van Barel (Katholieke Universiteit Leuven) Orthogonal functions and inverse eigenvalue problems
16h 00 - 16h 30	COFFEE BREAK and poster session
16h 30 - 17h 00	Lun Zhang (KU Leuven) Ladder operators and differential equations for multiple orthogonal polynomials
17h 00 - 17h 30	Maksym Derevyagin (TU Berlin) From CMV matrices to Jacobi matrices
17h 30 - 18h 00	Abey Lopez (KU Leuven) Multiple orthogonal polynomials arising in the normal matrix model with a quartic potential
20h00	CONFERENCE DINNER

Time	**Friday 29 June**
9h 00 – 9h 50	Guillermo Lopez-Lagomasino (Univ. Carlos III, Madrid) Direct and inverse results on the convergence of row sequences of Hermite-Padé approximation
10h 00– 10h 30	COFFEE BREAK
10h 30– 11h00	Stanislas Kupin (Université Bordeaux 1) On the growth of the polynomial entropy integrals for measures from the Szegő class
11h 00– 11h30	Miroslav Pranic (University of Banja Luka) Recurrence relations for orthogonal rational functions
11h 30– 12h00	Mirta Castro Smirnova (Universidad de Sevilla) On a paper by Karlin and McGregor
12h 00 - 13h 45	Lunch (Restaurant Universitaire Charles Barrois)
14h00 – 14h50	Juliette Leblond (APICS, INRIA Sophia Antipolis) Best approximation in generalized Hardy classes, application to inverse problems for elliptic PDE
15h 00 – 15h 50	Tom Claeys (Univ. Louvain La Neuve) Random matrices with equi-spaced external source
16h 00 - 16h 30	COFFEE BREAK and Poster Session
16h 30 - 17h 00	Mohamed Jalel Atia (Faculté des Sciences de Gabes) An explicit formula for the linearization coefficients of Bessel polynomials II
17h 00 - 17h 30	Ana Filipa Loureiro (Univ. Porto) The Kontorovich-Lebedev transform as a map between d-orthogonal polynomials
17h 30 - 18h 00	Amilcar Branquinho (Universidade do Coimbra) On the full Kostant Toda system and the discrete Korteweg de Vries equations

Abstracts of talks

Two completeness problems in the Paley-Wiener space

Alexander Borichev
Laboratoire d'Analyse, Topologie, Probabilités, Université Aix-Marseille

We study two completeness problems associated with exponential systems in the Paley-Wiener space. One is the hereditary completeness or the spectral synthesis problem going back to the 1970-s. Another one is motivated by a recent restricted shift problem by M.Carlsson and C.Sundberg. This is a joint work with A.Baranov and Yu.Belov.

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On the full Kostant Toda system and the discrete Korteweg de Vries equations

Amilcar Branquinho
Universidade do Coimbra

The relation between the solutions of the full Kostant Toda lattice and the discrete KdV equation is analyzed. A method for constructing solutions of these systems is given. As a consequence of the matrical interpretation of this method, the transform of Darboux is extended for general Hessenberg banded matrices.

Authors: Dolores Barrios Rolania; Ana Foulquié Moreno; Amilcar Branquinho

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On a paper by Karlin and McGregor

Mirta Castro Smirnova
Universidad de Sevilla

The seminal paper by S. Karlin and J. McGregor "Random walks" (1959) connects orthogonal polynomials and birth-and-death processes. Explicit results for the orthogonality measure and the orthogonal polynomials are given in two cases resulting from special relations among the parameters controlling the boundary condition at the origin. Here we allow for a general set of parameters and give the corresponding measure and orthogonal polynomials. This is joint work with Alberto Grünbaum.

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Random matrices with equi-spaced external source

Tom Claeys
Université Louvain la Neuve

I will discuss random matrix ensembles with a full rank external source. The eigenvalues of the external source are equally spaced on an interval. I will set up a Riemann–Hilbert problem for the associated multiple orthogonal polynomials and explain how asymptotics for this problem can be obtained. The limiting mean eigenvalue distribution of the model will be described in terms of an equilibrium problem, bulk and edge universality will be discussed, as well as asymptotics for the multiple orthogonal polynomials. This is based on joint work in progress with Dong Wang.

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From CMV matrices to Jacobi matrices

Maksym Derevyagin
TU Berlin

In the framework of theory of linear pencils, we introduce a new map from polynomials orthogonal on the unit circle to polynomials orthogonal on the real line by making use of the structure of CMV matrices. In fact, the Delsarte-Genin map can be considered as a particular case of the proposed one. Another interesting fact is that our map transforms Jacobi polynomials on the unit circle to the so called big -1 Jacobi polynomials on the real line.

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An explicit formula for the linearization coefficients of Bessel polynomials II

Mohamed Jalel Atia
Faculté des Sciences de Gabes

We prove a single sum formula for the linearization coefficients of the Bessel polynomials. In three special cases we show that this formula reduces indeed to either Atia and Zeng's formula (Ramanujan Journal, to appear) or Berg and Vignat's formulas in their proof of the positivity results about these coefficients (Constructive Approximation, **27** (2008), 15-32). As a bonus we also obtain a generalization of the formula for the linearization coefficients of the Bessel polynomials under Askey's terminology.

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On the growth of the polynomial entropy integrals for measures from the Szegő class

Stanislas Kupin
Université Bordeaux 1

Let σ be a probability Borel measure on the unit circle $\mathbb{T}$ and $\{\phi_n\}$ be the orthonormal polynomials with respect to σ. One says that σ is a Szegő measure, if it has an arbitrary singular part σ_s, and $\int_{\mathbb{T}} \log \sigma' dm > -\infty$, where σ' is the density of the absolutely continuous part of σ, m being the normalized Lebesgue measure on $\mathbb{T}$. The entropy integrals for ϕ_n are defined as

$$\epsilon_n = \int_{\mathbb{T}} |\phi_n|^2 \log |\phi_n| d\sigma.$$

One can see that $\epsilon_n = \overline{o}(\sqrt{n})$. In this talk, we'll explain that this estimate is sharp (over a subsequence).

The talk is based on a joint work with S. Denisov from U. of Wisconsin, Madison.

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Best approximation in generalized Hardy classes, application to inverse problems for elliptic PDE

Juliette Leblond
APICS, INRIA Sophia Antipolis

In domains of the complex plane, generalized analytic functions together with associated Hardy classes are related to some elliptic partial differential equations involved in physical models.
We will discuss the behaviour of solutions to Dirichlet (direct) or Cauchy type (inverse) problems for the conductivity equation: $\nabla \cdot (\sigma \nabla u) = 0$ (and for a related static Schrödinger equation) with smooth enough dilation coefficient σ, in domains of $\mathbb{C}$. These coincide with real parts of solutions to

the conjugate Beltrami equation: $\bar{\partial} f = \nu \, \overline{\partial f}$, with $\nu = (1-\sigma)/(1+\sigma)$, that defines a class of generalized holomorphic (or pseudo-analytic) functions (already studied by Bers and Vekua around 1950). Hardy boundedness conditions are in accordance with smoothness assumptions on available boundary Dirichlet or Neumann data.
Robust formulations of Cauchy inverse data transmission problems from partial overdetermined boundary data are provided by best constrained approximation issues in such Hardy classes. Solutions to these bounded extremal problems will be described, with constructive recovery schemes in the Hilbertian setting and in particular situations, which involve Toeplitz operators. The related classes of PDE include Laplace equations and particular conductivity equations, that arise for instance in physical applications related to plasma confinment, with magnetic data, from (quasi-static) Maxwell equations. Joint work with L. Baratchart, Y. Fischer, J.R. Partington

[1] K. Astala, L. Päivärinta, Calderón's inverse conductivity problem in the plane, *Ann. of Math.* (2) 16, no. 1, 265–299, 2006.

[2] L. Baratchart, J. Leblond, S. Rigat, E. Russ, Hardy spaces for the conjugate Beltrami equation in smooth domains of the complex plane, *J. Funct. Anal.*, 259(2), 384-427, 2010.

[3] Y. Fischer, *Approximation des des classes de fonctions analytiques généralisées et résolution de problèmes inverses pour les tokamaks*, PhD Thesis, Univ. Nice Sophia Antipolis, 2011.

[4] Y. Fischer, J. Leblond, J.R. Partington, E. Sincich, Bounded extremal problems in Hardy spaces for the conjugate Beltrami equation in simply connected domains, *Appl. Comp. Harmonic Anal.*, 31, 264-285, 2011.

[5] V.V. Kravchenko, *Applied Pseudoanalytic Function Theory*, Frontiers in Math., Birkhäuser Verlag, 2009.

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Large deviation in certain vector energy settings

Norman Levenberg
Indiana University, Bloomington, USA

For n nonpolar compact sets $K_1, ..., K_n \subset \mathbf{C}$, weights $Q_1, ..., Q_n$ and a positive semi-definite interaction matrix $C = [c_{ij}]_{i,j=1,...,n}$, we define natural discretizations of the weighted energy

$$E(\mu) = \sum_{i,j=1}^{n} c_{ij} I(\mu_i, \mu_j) + 2 \sum_{j=1}^{n} \int_{K_j} Q_j d\mu_j$$

of an $n-$tuple of probability measures $\mu = (\mu_1, ..., \mu_n) \in \mathcal{M}(K_1) \times \cdots \times \mathcal{M}(K_n)$. We have an $L^\infty-$type discretization $W(\mu)$ and an L^2-type discretization $J(\mu)$ defined using a fixed measure ν. This leads to a large deviation principle for a canonical sequence $\{\sigma_k\}$ of measures on $\mathcal{M}(K_1) \times \cdots \times \mathcal{M}(K_n)$ if ν is a strong Bernstein-Markov measure. This is joint work in progress with Tom Bloom and Franck Wielonsky.

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Multiple orthogonal polynomials arising in the normal matrix model with a quartic potential

Abey Lopez
KU Leuven

In this talk we analyze certain polynomials that satisfy non-Hermitian orthogonality conditions with respect to three exponential-type weights supported on unbounded contours in the complex plane. These polynomials arise from a recent approach of P. Bleher and A. Kuijlaars to the study of the normal matrix model. We present results describing the zero asymptotic distribution and strong asymptotic behavior of these polynomials. This is a joint work with A. Kuijlaars.

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Direct and inverse results on the convergence of row sequences of Hermite-Padé approximation

Guillermo Lopez-Lagomasino
Universidad Carlos III, Madrid

We present a Montessus de Ballore type theorem on the convergence of type II Hermite-Padé approximants of an analytic vector function. We show that under appropriate assumptions the zeros of the common denominator polynomials converge to "poles" of the vector function with geometric rate. Reciprocally, if the zeros of the common denominator polynomials converge with geometric rate to a system of points, we show that there exists a vector analytic function whose poles are precisely the points singled out. The direct result is an improvement of a theorem by Saff/Graves Morris and the inverse is an analogue of a well known theorem by A.A. Gonchar of the convergence relative to rows of Padé approximation. This is joint work with Bernardo de la Calle and Junot Cacoq.

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The Kontorovich-Lebedev transform as a map between d-orthogonal polynomials

Ana Filipa Loureiro
Universidade do Porto

The action of index integral transforms on d-orthogonal polynomial sequences will be at the centre of the discussion. A particular attention will be devoted to the most simple transform - the Kontorovich-Lebedev (KL). After a slight modification, this operator becomes an automorphism on the vector space of polynomials, which, for instance, allows the passage of the

2-orthogonal polynomial sequence of Laguerre type into the Continuous Dual Hahn polynomials. Finally, while seeking all the orthogonal polynomial sequences whose KL-transform is a d-orthogonal sequence, we realize they must be semiclassical.

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Recurrence relations for orthogonal rational functions

Miroslav Pranic
University of Banja Luka

It will be showed that orthogonal rational functions with preassigned poles satisfy short recurrence relations, analogous to the three-term recurrence relation for orthogonal polynomials. The number of terms in these recursions depends both on the number of distinct poles and on the order in which the poles enter in the sequence of orthogonal rational functions. Their applications to rational Gauss quadrature and to evaluation of matrix functions will be presented.

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Highlights of ATAP

Nick Trefethen
Oxford University

I have just completed a new book, "Approximation Theory and Approximation Practice", to be published by SIAM in about six months. This talk will describe some of the highlights of the book, both mathematical and otherwise.

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Orthogonal functions and inverse eigenvalue problems

Marc Van Barel
Department of Computer Science, Katholieke Universiteit Leuven

Orthogonal polynomials on the real line satisfy a three term recurrence relation. This relation can be written in matrix notation by using a tridiagonal matrix. Similarly, orthogonal polynomials on the unit circle satisfy a Szegő recurrence relation that corresponds to an (almost) unitary Hessenberg matrix. It turns out that orthogonal rational functions with prescribed poles satisfy a recurrence relation that corresponds to diagonal plus semiseparable matrices. This leads to efficient algorithms for computing the recurrence parameters for these orthogonal rational functions by solving corresponding linear algebra problems. In this talk we will study several of these connections between orthogonal functions and matrix computations and give some numerical examples illustrating the numerical behaviour of these algorithms.

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Ladder operators and differential equations for multiple orthogonal polynomials

Lun Zhang
Katholieke Universiteit Leuven

In this talk, we are dealing with the ladder operators and associated compatibility conditions for the type I and the type II multiple orthogonal polynomials. These ladder equations extend known results for orthogonal polynomials and can be used to derive the differential equations satisfied

by multiple orthogonal polynomials. Our approach is based on Riemann-Hilbert problems and the Christoffel-Darboux formula for multiple orthogonal polynomials, and the nearest-neighbor recurrence relations. As an illustation, we will also discuss several explicit examples.

This is a joint work with Galina Filipuk and Walter Van Assche.

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Posters

On Durrmeyer type modification of generalized Baskakov operators

Akif Barbaros Dikmen
Istanbul University

In the present poster we introduce a new Durrmeyer type modification of generalized Baskakov operators. We establish certain direct theorems in terms of the modulus of continuity of second order, the elements of Lipschitz-type space and the usual modulus of continuity.

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L_1C^1 approximation for large scattered datasets.

Laurent Gajny
ENSAM Lille

We address the problem of approximating data points by regular L_1-spline polynomial curves or surfaces of smoothness C^1. To obtain a C^1 cubic approximation curve, we have defined an iterative minimization method using five-data-point-sliding window. This procedure is extended to create C^1-continuous L_1 bicubic spline surfaces on images. The use of L_1-norm produces curve and surface approximations without oscillations whereas the data are noisy. By using sliding windows, the computational complexity of the algorithm for this initial non linear minimization algorithm becomes linear.

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Convergence of barycentric rational interpolation for analytic functions

Stefan Guettel
Oxford University

Polynomial interpolation to analytic functions can be very accurate, depending on the distribution of the interpolation nodes. However, in equispaced nodes and the like, besides being badly conditioned, such interpolation schemes may fail to converge even in exact arithmetic. Linear barycentric rational interpolation with the weights presented by Floater & Hormann (2007) can be viewed as blended polynomial interpolation and often yields better approximation in such cases. With the help of logarithmic potential theory we derive asymptotic convergence results for these interpolants, and we suggest how to choose the so-called blending parameter in order to observe fast and stable convergence, even in equispaced nodes where stable geometric convergence is provably impossible. This is joint work with Georges Klein from the University of Fribourg.

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Orthogonal polynomials with respect to some exponential weights in the complex plane

Nele Lejon
KU Leuven, Belgium

We study orthogonal polynomials with respect to some exponential weights $e^{-nV(z)}$, along a curve Γ in the complex plane. The monic polynomials of interest $P_n(z)$ have to satisfy:

$$\int_\Gamma P_n(z) z^k e^{-nV(z)} dz = 0 \qquad \text{for } k = 0, \ldots, n-1.$$

The zeroes of the orthogonal polynomials converge to a curve satisfying the S-property. For the potential $V(z) = -\frac{iz^3}{3}$, it is shown by Deano, Huybrechs and Kuijlaars that this curve consists of a single cut. This makes the zeroes particularly useful to study the zeroes as quadrature points for numerical approximation of highly oscillatory integrals.
Here we discuss two variants that are also of interest in that application. The first potential is:

$$V(z) = -\frac{iz^5}{5}.$$

In this case there are two families of orthogonal polynomials. The zeroes of each set converge to a single cut curve with the S-property. The second potential we study is:

$$V(z) = -\frac{iz^3}{3} + iKz, \qquad \text{if } K > 0.$$

We admitted a linear perturbation to the previously studied potential. It turns out that there is a critical value K^* for the perturbation constant K so that the zeroes converge to a single cut curve if $K < K^*$. In the critical case, where $K = K^*$ the curve with the S-property has a corner point. If the pertubation is supercritical, there is no curve with the S-property consisting of a single cut.

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Local analysis for Nikishin system at critical regime

Vladimir Lysov
Moscow, Russia

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Filtering the Tau Method with Chebyshev-Padé Approximants

Joao Matos
Porto

The Tau method to approximate the solution of a linear ordinary differential problem is based on solving a system of linear algebraic equations, obtained by imposing certain conditions for the minimization of the residual. For nonlinear differential problems the Tau approximation of the solution is usually obtained using a sequence of tau approximations of a linearized version of the nonlinear differential equations.

We have implemented a version of this method, obtaining a sequence of polynomials and, with their coefficients, we construct a sequence of Chebyshev-Padé approximants to the solution of the differential problem. Numerical examples suggest that this postprocess allows to improve the accuracy, expand the domain of validity given by the Tau approximation and localize the solution singularities of the differential equation.