Calculating Rational Best Approximants on $(-\infty, 0]$

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In many applications one needs rational approximations on the negative axis $\mathbb{R}_-$ of the exponential function or a function of similar type. In our talk we consider rational best approximants $r^*_{n,n+k} = r^*_{n,n+k}(f, \mathbb{R}_-; \cdot) \in R_{n,n+k}$ of a given function f on $\mathbb{R}_-$ in the uniform norm.

After a short review of characteristic properties of such approximants (the $'1/9'-$problem and related asymptotic considerations), we concentrate on numerical methods for their calculation. In the literature one finds two approaches for practical use: One is based on AAK approximation after the problem has been transformed from $\mathbb{R}_-$ onto the unit circle, and the other one has the Remez algorithm as its core piece.

We will describe a new variant of the algorithm. One of its main features is the exploitation of structural properties of the rational best approximants $r^*_{n,n+k}$, another one is the use of specific knowledge of the asymptotic behaviour of the error function.