Well and Ill Posed Inversion Problems for Matrix Algebras

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Given a matrix or an operator T with eigenvalues λ_j, $j \geq 1$, we say that the inversion problem for functions (polynomials) in T is well posed if the norm of $f(T)^{-1}$ can be bounded in terms of $min_j|f(\lambda_j)| = \delta > 0$ for every f such that $\|f(T)\| \leq 1$.

(1) We give a criterion for such a well posedness in terms of the so-called (Carleson-like) Weak Embedding Property for $\sigma = (\lambda_j)$ and give many examples of the spectra satisfying (or not) this condition (joint result with P.Gorkin and R.Mortini).

(2) Moreover, given a constant $0 \leq \delta_1 \leq 1$ we show that there exist (infinite) spectra σ such that the above inversion problem is well posed for all δ with $\delta_1 < \delta \leq 1$ and ill posed for all δ with $0 < \delta < \delta_1$ (joint result with V.Vasyunin).

(3) Finally, we use these results for disproving an analog of the paving conjecture for Hilbert space unconditional block-bases (for Hilbert space unconditional bases, this conjecture is equivalent to the famous Kadison-Singer problem).