Non-intersecting squared Bessel paths and multiple orthogonal polynomials for modified Bessel weights

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We study a model of n non-intersecting squared Bessel processes in the confluent case: all paths start at time $t = 0$ at the same positive value $x = a$, remain positive, and are conditioned to end at time $t = T$ at $x = 0$. In the limit $n \to \infty$, after appropriate rescaling, the paths fill out a region in the tx-plane that we describe explicitly. In particular, the paths initially stay away from the hard edge at $x = 0$, but at a certain critical time t^* the smallest paths hit the hard edge and from then on are stuck to it. For $t \neq t^*$ we obtain the usual scaling limits from random matrix theory, namely the sine, Airy, and Bessel kernels. A key fact is that the positions of the paths at any time t constitute a multiple orthogonal polynomial ensemble, corresponding to a system of two modified Bessel-type weights. As a consequence, there is a 3×3 matrix valued Riemann-Hilbert problem characterizing this model, that we analyze in the large n limit using the Deift-Zhou steepest descent method. There are some novel ingredients in the Riemann-Hilbert analysis that are of independent interest.

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