Rational interpolation to exponential-like functions on elliptic lattices

Alphonse Magnus
Université de Louvain-La-Neuve, Belgique
3 ièmes journées Approximation 15-16 Mai Lille

A function is called exponential-like with respect to a difference operator if it satisfies

$$Df(x) = a[f(\psi(x)) + f(\phi(x))],$$

where the (divided) difference operator is

$$Df(x) = [f(\psi(x)) - f(\phi(x))]/[\psi(x) - \phi(x)].$$

The functions ϕ and ψ define the setting of the theory, from the most elementary choice $(x, x+h)$ to forms $R(x)$ plus or minus square root of $S(x)$, where R and S are rational functions of degrees up to 2 and 4. Remark that the difference equation is a symmetric combination of the two conjugate algebraic functions ϕ and ψ. The difference equation is also a recurrence relation on a lattice built from $y(n) = \phi(x(n)), \quad y(n+1) = \psi(x(n))$, from which $x(n+1)$ is found through $y(n+1) = \phi(x(n+1))$. When the degrees of$R$ and S are 2 and 4, we get a so-called elliptic lattice, or grid, as $x(n)$ and $y(n)$ appear to be elliptic functions of n (Baxter, Spiridonov, Zhedanov). The exponential-like function of above is interpolated on such a lattice $y(0), y(1), ...$ by rational functions with poles on a well-chosen sequence $y'(0), y'(1), ...$