Orthogonal polynomials and the interlacing of zeros

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It is a well-known classical result that the zeros of orthogonal polynomials of adjacent degree are interlacing. We discuss the extent to which the interlacing of zeros can be proved in many different situations where orthogonal polynomials, particularly classical orthogonal polynomials, are involved.

The zeros of polynomials of the same or adjacent degree from different orthogonal sequences may or may not interlace and we give proofs (or counter-examples where appropriate) for the one-parameter families of Laguerre and Gegenbauer polynomials, as well as the two-parameter family of Jacobi polynomials. In these cases, the different sequences are generated by allowing the parameter(s) to vary continuously and/or in integer steps.

We review related results for the interlacing of zeros of linear combinations of classical orthogonal polynomials, including those that arise as a result of quasi-orthogonality.

We conclude with a discussion of an open question raised by F Marcellan in 2007 concerning the conditions under which linear combinations of orthogonal polynomials from distinct orthogonal sequences are themselves orthogonal.