

Hardy spaces of the real Beltrami equation

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We introduce Hardy spaces of solutions to the so-called real Beltrami equation in the disk: $\bar{\partial} f = \nu(z)\bar{\partial} \bar{f}$, where $-1 + \varepsilon < \nu < 1 - \varepsilon$ and ν is Lipschitz continuous. Dwelling on some work by Bers and Nirenberg on pseudo-analytic functions, we prove the L^p boundedness of the conjugation operator mapping u to v if $f = u + iv$ on the circle, and f has real mean. We also show the density of such functions on strict subarcs of the circle. This allows us to consider bounded extremal problems in such classes of functions. A motivation for such a study comes from the fact that the compatibility condition for $f = u + iv$ to solve the Beltrami equation is that $\operatorname{div}(\sigma \nabla u) = 0$ where $\sigma = (1 - \nu)/(1 + \nu)$. This way, extremal problems arising for solutions to diffusion equations can be recast in terms of pseudo-analytic functions. We exemplify this in the case of an inverse boundary problem arising in plasma control.